Modified Parton Branching Model for Multi-Particle Production in Hadronic Collisions

Application to SUSY Particle Branching

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I would like to dedicate this thesis to my loving parents …

Declaration

I hereby declare that this thesis is my original work and it has been written by me in its entirety. I have duly acknowledged all the sources of information which have been used in the thesis.

This thesis has also not been submitted for any degree in any university previously.

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Abstract

The stochastic branching model of multi-particle productions in high energy collision has theoretical basis in perturbative QCD, and also successfully describes the experimental data for a wide energy range. However, over the years, little attention has been put on the branching model for supersymmetric (SUSY) particles.

In this thesis, a stochastic branching model has been built to describe the pure supersymmetric particle jets evolution. This model is a modified two-phase stochastic branching process, or more precisely a two phase Simple Birth Process plus Poisson Process. The general case that the jets contain both ordinary particle jets and supersymmetric particle jets has also been investigated. We get the multiplicity distribution of the general case, which contains a Hypergeometric function in its expression. We apply this new multiplicity distribution to the current experimental data of pp collision at center of mass energy $\sqrt{s} = 0.9, 2.36, 7$ TeV. The fitting shows the supersymmetric particles haven't participate branching at current collision energy.

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Chapter 1

Introduction

The discovery of Higgs Boson at Large Hadron Collider (LHC) in 2012 confirms the last unverified part of Standard Model. Standard Model describes the three kinds of interactions between elementary particles, electromagnetic interaction, weak interaction and strong interaction. The elementary particles include matter constituent particles (quarks and leptons) and interaction mediator particles (photon, $W^{\pm}$ Z^0 bosons, gluons). The electromagnetic interaction can be described perfectly by Quantum Electrodynamics (QED), with extremely accurate quantitative predictions. The weak interaction is united with electromagnetic interaction into electroweak interaction, which is depicted by electroweak theory. The strong interaction is described by Quantum Chromodynamics (QCD), and particles with color charge (analog of electric charge in QED) undergo strong interaction. Electroweak theory and QCD are two main blocks of Standard Model.

To verify the Standard Model, various particle colliders and cosmic ray detectors have been built to create and analyze reactions governed by electroweak theory and QCD. The comparison of observables from experiment measurements and theory predictions can validate the Standard Model theory or lead to beyond Standard Model physics. Final state charged-particle multiplicity distribution (MD) is one of the simplest observables in high energy particle collisions, which contains information of underlying dynamical and statistical mechanism of multi-particle production. It is also the topic of this thesis.

Besides the elaborate experimental analysis to extract the measurement value of the charged particle multiplicity distribution [21, 24], the precise quantitative theory of the multi-particle production process is of particular significance. The particle collisions involve several stages and various sub-processes, for different sub-processes the theoretical models or approximation methods are different. In this thesis, hadronic collision is our focus.

1.1 Hadronic Collision

For Hadronic collision (pp or $p\bar{p}$ collisions), the whole process can be divided into three stages: Before Scattering, Scattering and After Scattering.

Before Scattering, the two protons (antiprotons) come towards each other. Here the protons are extended objects with quarks and gluons inside, and can be depicted by parton density function (PDF)

$$G_h^a(x, Q^2) \tag{1.1}$$

which represents the probability density of finding a parton (quark or gluon) of type a inside the hadron (proton/antiproton) h, this parton carries x fraction of longitudinal momentum of the hadron (proton/antiproton) h. The Q^2 is the energy scale of the interaction to probe the PDF. On the way to the collision point, each parton inside the hadrons can radiate new particles or split into two particles, this is called initial state shower.

Scattering consists of hard scattering, semi-hard scattering and soft scattering. Semi-hard and soft scattering are combined to be defined as underlying events.

In hard scattering, two partons collide and produce outgoing partons, with large momentum transfer in this process. It is possible to have more than one hard scattering in one collision event [23], which is called multi-parton interaction. The hard scattering can be described by perturbative QCD (pQCD) quite well. Hard scattering will produce two or more jets [1] with large transverse momentum p_T, but it only occupies a small section of the total cross section or multiplicity contribution for current collision energy.

The other partons in the two hadrons (proton/antiproton) undergo semi-hard and soft scattering, with low momentum transfer. In some model [18], one can extend the pQCD calculation to semi-hard scattering (semi-hard jets defined as jet transverse momentum $1\text{GeV} \ll |p_T| \ll \sqrt{s}$, $\sqrt{s}$ the collision energy). As for soft scattering, which cannot be described by pQCD, is described by soft hadron theory. Soft scattering contains two kinds: elastic scattering and inelastic scattering. For elastic scattering, the two hadrons emerge intact after the collision, and they exchange a Pomeron [2]. No new particles are produced in elastic scattering. For inelastic scattering, new particles are produced. According to whether the two hadrons exchange color charge or not, inelastic scattering is divided into non-diffractive and diffractive scattering. Non-diffractive (ND) scattering, with color charge exchanged, produce particles homogeneously distributed in space, so there is no rapidity gap for ND scattering. Also ND scattering produces more particles than diffractive scattering, and dominates *pp* and

[1] High energy free quark or gluon (carry color charge), create particles around them, form a spray of collimated particles, called jet.

[2] The Pomeron can be interpreted as colorless and flavorless combination of multiple gluons.

$p\bar{p}$ collisions. In diffractive scattering, two hadrons exchange one or more colorless Pomerons. According to the event shape, or multiplicity distribution over pseudorapidity η (refer to footnote 4), people define single diffractive, double diffractive and central diffractive scattering, etc.

One may find the parton interpretation of hard scattering is quite different from the Pomeron interpretation of soft scattering, this is due to the historical reason. The Pomeron was introduced in 1961, before the proposal of quark model (1964). What's more, the soft hadron physics remains an incomplete and qualitative theory till now. There is much to discover in it.

After scattering, the outgoing partons radiate particles or split into two particles, this is the final state shower, also called parton branching (the central topic of this thesis). The model for this process will be discussed in detail in the following text. When the virtuality[3] of the partons reaches Q_0^2– the non-perturbative threshold, the partons start to form colorless hadrons. This process is called hadronization. For Hadronization, the Local Parton-Hadron Duality (LPHD) [1] was proposed, which states that the spectra of parton before hadronization is proportional to spectra of hadrons after hadronization. The LPHD is based on the locality of parton branching and hadronization in configuration space.

1.2 Multiplicity Distribution

The probability p_n of having n particles in the final state of particle collisions has a distribution over n. The probability distribution p_n is called the Multiplicity Distribution (MD) in particle physics.

[3]The virtuality is defined as $t = p^\mu p_\mu - m_0^2$, which measures how much a particle is off shell.

KNO scaling [17] starts the widespread theoretical interests for multiplicity distribution. It states the multiplicity distribution $p_n(\sqrt{s})$ has the form at large center of mass energy $\sqrt{s}$:

$$\bar{n}p_n(\sqrt{s}) = \psi(\frac{n}{\bar{n}}) \tag{1.2}$$

and the mean multiplicity expression as $\bar{n} = a \ln \sqrt{s} + b$. The other prediction of KNO scaling is about the C_q moment

$$C_q = \frac{\overline{(n^q)}}{(\bar{n})^q} = const.\text{(for all } q) \tag{1.3}$$

KNO scaling is an elegant theoretical result, simple in the expression form. Later research all tried to verify or veto the KNO scaling: Theoretical model checked whether the analytical expression of multiplicity distribution follow the KNO scaling form or not [5, 9]. Experimental analyses plotted the $\psi(\frac{n}{\bar{n}})$ and C_q to check the validation or violation of KNO scaling[24]. KNO scaling was found to be broken by the UA5 collaboration in $p\bar{p}$ collisions at $\sqrt{s} = 200$ GeV [26] and $\sqrt{s} = 540$ GeV [25].

UA5 collaboration also found that the Negative Binomial Distribution(NBD) can describe the data up to $\sqrt{s} = 540$ GeV. The Negative Binomial Distribution contains two parameters $\bar{n}$ and k:

$$p_{\mathrm{NBD}}(n; \bar{n}, k) = \left[\frac{\bar{n}}{k+\bar{n}}\right]^n \left[\frac{k}{k+\bar{n}}\right]^k \frac{\Gamma(n+k)}{\Gamma(k)\Gamma(n+1)} \tag{1.4}$$

NBD proposed by Giovannini and Van Hove [13] can be understood in the stochastic clan model, which groups the particles of same ancestor as a clan, but the underlying physics is still not so clear. UA5 latter found at $\sqrt{s} = 900$ GeV, single

NBD can only describe the data for small pseudorapidity[4] intervals η . In larger pseudorapidity intervals, the shoulder structure appeared cannot be described by single NBD. Giovannini and Ugoccioni [11] proposed a two component NBD distribution to describe the shoulder structure, soft component stands for events without mini-jets [5] and semi-hard component for events with mini-jets.

Giovannini's NBD distribution and the branching interpretation of particle production are based on the considerations as follows. If we assume the Local Parton Hadron Duality(LPHD) and ignore the initial state shower, the process of multi-parton production can be reduced from whole collision to final state shower after the hard, semihard and soft scattering. For hard scattering, the final state shower is the jet evolution, which can be calculated via perturbative QCD (pQCD). The idea of relating jet evolution with stochastic branching process was firstly proposed by Orfanidis and Rittenberg [19] and Polyakov [20]. Later Konishi *et al.* [18] used this idea and extended the pQCD calculation to semi-hard region (semi-hard jets), with jet transverse momentum $1\text{GeV} \ll |p_T| \ll E = \sqrt{s}$. Konishi *et al.* [18] derived two equations satisfied by multiplicity generating functions of quarks and gluons, and found they are exactly the stochastic equations for population of two spices. Giovannini [10] got the same equations via Markov branching process construction. In this case, the hard and semi-hard jets contribution to multiplicity distribution can be calculated by stochastic approach. For soft interaction part, one assumes the branching process is extendable to soft region, where no calculation methods exist either from pQCD or non-perturbative theory. The investigation on the branching mechanism of multi-particle production may give some insights on the underlying physics for soft

[4]Pseudorapidity $\eta = -\ln[tan(\theta/2)]$ is a measure of the angle θ between particle and beam axis. The Larger the angle θ, the smaller the pseudorapidity η.

[5]Defined by UA5 : groups of particles having total transverse energy larger than 5 GeV.

interaction and connection between soft hadron physics and hard hadron physics [18].

Following Giovannini's interpretation, different multiplicity distributions are proposed and investigated. In Giovannini's paper [10], he himself solved the two branching equations under different approximations, and got the multiplicity distributions for single quark- and single gluon- jets. Durand and Sarcevic [9] investigated pure gluon bremsstrahlung solution and Negative Binomial Distribution (NBD) solution of the branching equation and its relation to KNO scaling. Chen and Hwa [5] proposed a geometric scaling model, which consists of Furry process [6] and impact parameter smearing. The multiplicity distribution is the convolution of the Furry distribution with an impact parameter dependent function. In a later paper, Hwa [15] considered the effect of mini-jets, and included the mini-jets production in the multiplicity distribution as "hard" component with jet virtuality smearing (analog of impact parameter smearing). Chew *et al.* [6] proposed a generalized multiplicity distribution (GMD), with arbitrary initial number of gluons and quarks. Chew also commented that gluons and quarks can stand for two kinds of particles, which indicate one can extend stochastic branching model to soft hadron interaction.

1.3 Outline of Thesis

Supersymmetry, as a promising candidate for beyond Standard Model physics, attracts much research interests, including both theoretical and experimental studies. But little effort has been made on the branching of supersymmetric particle

[6]Furry process is the Simple birth process in stochastic process, it's one kind of branching process.

jets[7], this thesis will try to establish a stochastic branching model for supersymmetric particle jets evolution. In addition, the general case of mixed jets (supersymmetric particle jets and ordinary parton jets) is also investigated.

The thesis is organized as follows: Chapter 2 demonstrates the modified stochastic branching model, accompanied by some introduction about mathematical concepts and several simple stochastic processes. In Chapter 3, we review the way to extract the branching properties of QCD jets, and discuss the evolution parameter in detail. Generalized Multiplicity Distribution (GMD) and its composition are also studied. Chapter 4 starts by some introduction about supersymmetry and its motivation. Then we use the modified stochastic branching model (introduced in Chapter 2) to describe the pure supersymmetric(SUSY) particle branching process. The more general case, SUSY particle plus ordinary parton initiated jets, has been investigated. We fit the multiplicity distribution of the general case to current experimental data. Conclusion and further remarks are presented in Chapter 5.

[7] There is some Monte Carlo simulations which incorporate the SUSY particle branching [3].

Chapter 2

Modified Stochastic Branching Model

Stochastic Process is a mathematical description of the evolution of a system, whose status is represented by random value at each time instant, for example the population of the system. It has wide application to various disciplines in science. Now we introduce some basic mathematical concepts used in stochastic process and some typical stochastic processes, before move on to description about our modified stochastic branching model. For more comprehensive elaboration of the stochastic process, one can refer to the book from Bailey [2] .

2.1 Basic Theory

When dealing with the populations of the system, generating function is widely used to simplify the handling of the process. The population of a system can only take an integer value n at time t, the probability of the population taking value n is $p_n(t)$, with probability normalization:

$$\sum_{n=0}^{\infty} p_n(t) = 1 \tag{2.1}$$

The probability generating function for time t is defined as:

$$P(x,t) = p_0 + p_1 x + p_2 x^2 + \cdots + p_n x^n + \cdots = \sum_{n=0}^{\infty} p_n(t) x^n \tag{2.2}$$

One can see from the definition that probability generating function contains all the information of the system at time t, i.e. all probabilities of system taking any value for population. One can get the probability from probability generating function,

$$p_n(t) = \frac{1}{n!} \frac{\partial^n P(x,t)}{\partial x^n} \tag{2.3}$$

The relevant moment generating function is defined as:

$$M(\theta,t) = p_0 + p_1 e^{\theta} + p_2 e^{2\theta} + \cdots + p_n e^{n\theta} + \cdots = \sum_{n=0}^{\infty} p_n(t) e^{n\theta} \tag{2.4}$$

Moments are generated by

$$\begin{aligned} M(\theta,t) &= \sum_{n=0}^{\infty} p_n(t) e^{n\theta} \\ &= \sum_{n=0}^{\infty} (1 + n\theta + \frac{(n\theta)^2}{2!} + \dots) p_n(t) \\ &= \sum_{n=0}^{\infty} p_n(t) + \theta \sum_{n=0}^{\infty} n p_n(t) + \frac{\theta^2}{2!} \sum_{n=0}^{\infty} n^2 p_n(t) \end{aligned} \tag{2.5}$$

the coefficient of Taylor series of $M(\theta, t)$ around $\theta = 0$ gives you the moment of any rank.

One can find the relation between moment generating function and probability generating function as

$$M(\theta, t) = P(e^{\theta}, t) \tag{2.6}$$

In stochastic process, there are some very simple types. Here we introduce two of them, the Poisson Process and the Simple Birth Process.

Poisson Process is characterized by the probability of the system producing a new normal individual[1] (use individual in the following text) in short time interval Δt, $\nu\Delta t$. Here the ν is a constant, independent of total population $n(t)$ and time t. Poisson Process is also called completely random process. The probability of having n individuals at time $t + \Delta t$ has two part

$$p_n(t + \Delta t) = p_{n-1}(t) \cdot \nu\Delta t + p_n \cdot (1 - \nu\Delta t) \tag{2.7}$$

one part is that $(n-1)$ individual at t, and then an individual is produced in time interval Δt. The other part comes from n individuals at time t, and no individual is produced in Δt, with probability of $1 - \nu\Delta t$.

We can find the differential equation that p_n satisfy by taking time interval to be infinitesimal,

$$\frac{dp_n}{dt} = \lim_{\Delta t \to 0} \frac{p_n(t + \Delta t) - p_n(t)}{\Delta t} = \nu[p_{n-1}(t) - p_n(t)] \tag{2.8}$$

[1]The normal individual means the component of the population, like a single microbe in a group of microbes.

By multiplying the equation with x^n and integrating over all n, we can get the equation for probability generating function $P(x,t)$:

$$\sum_{n=0}^{\infty} \frac{dp_n(t)}{dt} x^n = \nu \sum_{n=1}^{\infty} p_{n-1}(t) x^n - \nu \sum_{n=0}^{\infty} p_n(t) x^n \tag{2.9}$$

$$\frac{\partial P(x,t)}{\partial t} = \nu x P(x,t) - \nu P(x,t) \tag{2.10}$$

By substituting $x = e^{\theta}$, we get the equation for moment generating function $M(\theta, t)$:

$$\frac{\partial M(\theta,t)}{\partial t} = \nu e^{\theta} M(\theta,t) - \nu M(\theta,t) \tag{2.11}$$

For Poisson Process, we can interpret it as that some immigrants[2] give birth to normal individuals in the whole system, and the number of immigrants is fixed. This interpretation will be useful in our modified stochastic model and SUSY particle branching process analyses.

For Simple Birth Process (Furry Yule Process), each individual in the system has the probability $\lambda \Delta t$ to give birth to a new individual in time interval Δt. For the whole system, the probability of producing a new individual in time interval Δt is $n\lambda\Delta t$, which depends on the population size $n(t)$ at time t. Probability of having n individuals at time $t + \Delta t$ is

$$p_n(t + \Delta t) = p_{n-1}(t) \cdot \lambda(n-1)\Delta t + p_n(t) \cdot (1 - \lambda n) \tag{2.12}$$

[2]The immigrant can be understand as the mutant in a group of microbes, the immigrant (mutant) can give birth to the normal microbe as the microbes in the group.

Following the same procedures as for Poisson Process, we can get the differential equation for probability $p_n(t)$

$$\frac{dp_n(t)}{dt} = \lambda(n-1)p_{n-1}(t) - \lambda n p_n(t) \tag{2.13}$$

for probability generating function $P(x,t)$:

$$\frac{\partial P(x,t)}{\partial t} = \lambda x^2 \frac{\partial P(x,t)}{\partial x} - \lambda x \frac{\partial P(x,t)}{\partial x} \tag{2.14}$$

for moment generating function $M(\theta,t)$:

$$\frac{\partial M(\theta,t)}{\partial t} = \lambda e^{\theta} \frac{\partial M(\theta,t)}{\partial \theta} - \lambda \frac{\partial M(\theta,t)}{\partial \theta} \tag{2.15}$$

Poisson Process and Simple Birth Process share some common properties: First, they are all Markov process, the population of the whole system in next time instant is independent of past history. Secondly, the development of population for both system can be depicted as family tree, as a branching process.

2.2 Our Modified Branching Model

After getting familiar with some simple stochastic processes, it's time to introduce our modified branching model. Our model is the combination of Simple Birth Process and Poisson Process. It is a branching process and consists of two phases. The process of our model is started by some immigrants, which means the initial population $n(t=0)$ of the system is zero. The two phases of our model are characterized by different probabilities of producing a new individual ν_1, ν_2 for Poisson Process, corresponding to the number or type of immigrants changes from phase one to phase two. We assume the transition from phase 1 to phase 2 occurs at time t_1. The Simple Birth Process stays unchanged for phase 1 and phase 2, with constant individual birth rate λ.

Following the standard procedure, we can find the differential equation for our model, only with different Poisson Process parameters v_1, v_2 for two phases. At first, the probability of having n individuals at time $t + \Delta t$ is:

$$p_n(t + \Delta t) = p_{n-1}(t) \cdot [\lambda(n-1) + v]\Delta t + p_n \cdot [1 - (\lambda n + v)\Delta t] \tag{2.16}$$

then we get the differential equation for $p_n(t)$:

$$\frac{dp_n(t)}{dt} = -\lambda n p_n(t) + \lambda(n-1)p_{n-1}(t) - v p_n(t) + v p_{n-1}(t) \tag{2.17}$$

differential equation for probability generating function $P(x,t)$:

$$\frac{\partial P(x,t)}{\partial t} = \lambda x^2 \frac{\partial P(x,t)}{\partial x} - \lambda x \frac{\partial P(x,t)}{\partial x} + v x P(x,t) - v P(x,t) \tag{2.18}$$

and differential equation for moment generating function $M(\theta,t)$:

$$\frac{\partial M(\theta,t)}{\partial t} = \lambda(e^\theta - 1)\frac{\partial M(\theta,t)}{\partial \theta} + v(e^\theta - 1) \tag{2.19}$$

When solving the equation, we prefer to work with the momentum generating function for simplicity. Now let's start to solve the equation for our two phase branching model.

2.2.1 Phase One

For phase one, time t lies in $[0, t_1]$. The stochastic branching equation for phase one reads :

$$\frac{\partial M(\theta,t)}{\partial t} = \lambda(e^{\theta}-1)\frac{\partial M(\theta,t)}{\partial \theta} + \nu_1(e^{\theta}-1) \tag{2.20}$$

We find the subsidiary equations are given by

$$\frac{dt}{1} = -\frac{d\theta}{\lambda(e^{\theta}-1)} = \frac{dM(\theta,t)}{\nu_1(e^{\theta}-1)M(\theta,t)} \tag{2.21}$$

First and second term give

$$\lambda t + \ln(1-e^{-\theta}) = const. \tag{2.22}$$

while second and third term result in

$$e^{(\nu_1/\lambda)\theta} M(\theta,t) = const. \tag{2.23}$$

The general solution of equation 2.20 is

$$e^{(\nu_1/\lambda)\theta} M(\theta,t) = \Psi[\lambda t + \ln(1-e^{-\theta})] \tag{2.24}$$

The initial condition is that initial population is zero: $M(\theta,0) = 1$. At $t = 0$, the expression above reads

$$e^{(\nu_1/\lambda)\theta} = \Psi[\ln(1-e^{-\theta})] \tag{2.25}$$

using substitution

$$u = \ln(1 - e^{-\theta}) \rightarrow e^{\theta} = \frac{1}{1 - e^{u}} \tag{2.26}$$

we can get

$$\Psi[u] = \left[\frac{1}{1 - e^{u}}\right]^{\nu_1/\lambda} \tag{2.27}$$

then for any time t lies in $[0, t_1]$, from equation 2.24 and equation 2.27 we know the general solution is

$$e^{(\nu_1/\lambda)} M(\theta, t) = \left[\frac{1}{1 - e^{\lambda t + \ln(1 - e^{-\theta})}}\right]^{\nu_1/\lambda} \tag{2.28}$$

The moment generating function is

$$M(\theta, t) = \left[\frac{1}{(1 - e^{\lambda t})e^{\theta} + e^{\lambda t}}\right]^{\nu_1/\lambda} \tag{2.29}$$

We can find the mean multiplicity $\bar{n}$ via equation 2.5

$$\bar{n} = \frac{\nu_1}{\lambda}(e^{\lambda t} - 1) \tag{2.30}$$

From equation 2.6, we can get probability generating function

$$P(x, t) = \left[\frac{1}{(1 - e^{\lambda t})x + e^{\lambda t}}\right]^{\nu_1/\lambda} \tag{2.31}$$

by equation 2.3 the probability distribution is

$$p_n(t) = (1 - e^{-\lambda t})^n e^{-\nu_1 t} \frac{\Gamma(n + \frac{\nu_1}{\lambda})}{\Gamma(\frac{\nu_1}{\lambda})\Gamma(n+1)} \tag{2.32}$$

This is the Negative Binomial Distribution (NBD). It can be rewritten in terms of $\bar{n}$ and $k = \nu_1/\lambda$

$$p_n(t) = \frac{\bar{n}^n k^k}{(\bar{n}+k)^{n+k}} \frac{\Gamma(n+k)}{\Gamma(k)\Gamma(n+1)} \tag{2.33}$$

2.2.2 Phase Two

Phase two starts from t_1, the initial condition for phase two is the phase one solution at t_1

$$M(\theta, t_1) = \left[\frac{1}{(1 - e^{\lambda t_1})e^{\theta} + e^{\lambda t_1}}\right]^{\nu_1/\lambda} \tag{2.34}$$

since all the information of the system are collected in the generating function.

The branching equation and subsidiary equations for phase two only differ from phase one by replacing ν_1 with ν_2:

$$\frac{\partial M(\theta,t)}{\partial t} = \lambda(e^{\theta} - 1)\frac{\partial M(\theta,t)}{\partial \theta} + \nu_2(e^{\theta} - 1) \tag{2.35}$$

Similarly as equation 2.22 and equation 2.23, we have

$$\begin{aligned} \lambda t + \ln(1 - e^{-\theta}) &= const. \\ e^{(\nu_2/\lambda)\theta} M(\theta,t) &= const. \end{aligned} \tag{2.36}$$

By two equations above, the general solution of phase two branching equation is

$$e^{(\nu_2/\lambda)\theta} M(\theta, t) = \Psi[\lambda t + \ln(1 - e^{-\theta})] \tag{2.37}$$

the expression above at initial time of phase two t_1 is (substitute equation 2.34)

$$e^{(\nu_2/\lambda)\theta} \left[\frac{1}{(1 - e^{\lambda t_1})e^{\theta} + e^{\lambda t_1}}\right]^{\nu_1/\lambda} = \Psi[\lambda t_1 + \ln(1 - e^{-\theta})] \tag{2.38}$$

using the substitution

$$u = \lambda t_1 + \ln(1 - e^{-\theta}) \to \frac{1}{1 - e^{u - \lambda t_1}} \tag{2.39}$$

we get

$$\Psi[u] = \left(\frac{1}{1 - e^{u-\lambda t_1}}\right)^{\nu_2/\lambda} \left[\frac{1}{(1 - e^{\lambda t_1})\dfrac{1}{1 - e^{u-\lambda t_1}} + e^{\lambda t_1}}\right]^{\nu_1/\lambda} \tag{2.40}$$

At any time $t \in (t_1, \infty)$, from equation 2.37 and equation 2.40 we get the general solution:

$$e^{(\nu_2/\lambda)\theta} M(\theta, t) = \left(\frac{1}{1 - e^{\lambda t + \ln(1-e^{-\theta}) - \lambda t_1}}\right)^{\nu_2/\lambda} \left[\frac{1}{\dfrac{(1 - e^{\lambda t_1})}{1 - e^{\lambda t + \ln(1-e^{-\theta}) - \lambda t_1}} + e^{\lambda t_1}}\right]^{\nu_1/\lambda} \tag{2.41}$$

The moment generating function is

$$M(\theta, t) = \frac{\left[(1 - e^{\lambda(t-t_1)})e^{\theta} + e^{\lambda(t-t_1)}\right]^{(\nu_1/\lambda)-(\nu_2/\lambda)}}{\left[(1 - e^{\lambda t})e^{\theta} + e^{\lambda t}\right]^{(\nu_1/\lambda)}} \tag{2.42}$$

From equation 2.5, we can know the mean multiplicity $\bar{n}$ for phase two:

$$\bar{n} = \frac{\nu_2}{\lambda}[e^{\lambda(t-t_1)} - 1] + \frac{\nu_1}{\lambda}[e^{\lambda t} - e^{\lambda(t-t_1)}] \tag{2.43}$$

The probability generating function can be easily gotten via equation 2.6

$$P(x,t) = \frac{\left[(1 - e^{\lambda(t-t_1)})x + e^{\lambda(t-t_1)}\right]^{(\nu_1/\lambda)-(\nu_2/\lambda)}}{\left[(1 - e^{\lambda t})x + e^{\lambda t}\right]^{(\nu_1/\lambda)}} \tag{2.44}$$

by equation 2.3, we obtain the probability distribution :

$$\begin{aligned} p_n(t) =& e^{\lambda(t-t_1)[(\nu_1/\lambda)-(\nu_2/\lambda)]}(1 - e^{-\lambda t})^n e^{-\nu_1 t} \frac{\Gamma(n + \frac{\nu_1}{\lambda})}{\Gamma(\frac{\nu_1}{\lambda})\Gamma(n+1)} \\ & \times {}_2F_1(\frac{\nu_2}{\lambda} - \frac{\nu_1}{\lambda}, -n, 1 - \frac{\nu_1}{\lambda} - n, \frac{e^{\lambda t} - e^{\lambda t_1}}{e^{\lambda t} - 1}) \end{aligned} \tag{2.45}$$

which contains an ordinary Hypergeometric Function ${}_2F_1(a, b, c, d)$.

2.2.3 Probability Distribution and Generating Function

After solving our modified stochastic branching model (two phase Poisson Process plus Simple Birth Process, initial population zero), we get the probability distribution and its generating function, using substitutions $k_1 = \nu_1/\lambda$, $k_2 = \nu_2/\lambda$:

$$\begin{aligned} p_n(t) =& e^{\lambda(t-t_1)(k_1-k_2)}(1 - e^{-\lambda t})^n (e^{-\lambda t})^{k_1} \frac{\Gamma(n + k_1)}{\Gamma(k_1)\Gamma(n+1)} \\ & \times {}_2F_1(k_2 - k_1, -n, 1 - k_1 - n, \frac{e^{\lambda t} - e^{\lambda t_1}}{e^{\lambda t} - 1}) \end{aligned} \tag{2.46}$$

$$P(x,t) = \frac{\left[(1 - e^{\lambda(t-t_1)})x + e^{\lambda(t-t_1)}\right]^{k_1-k_2}}{\left[(1 - e^{\lambda t})x + e^{\lambda t}\right]^{k}_{1}} \tag{2.47}$$

Here the ν_1 (ν_2) denotes the probability $\nu_1\Delta t$ ($\nu_2\Delta t$) of Poisson Process to produce a new individual in time interval Δt of phase one (or two). The λ denotes the $\lambda\Delta t$ as individual birth rate and $\lambda n\Delta t$ as the probability for whole population to produce a new individual in Simple Birth Process for two phases.

Our modified stochastic model will be used to describe the pure supersymmetric particle branching, in Chapter 4, with direct usage of the probability distribution and generating function derived in this chapter.

Chapter 3

QCD Jets Branching Process

Before we apply the modified stochastic branching model (discussed in Chapter 2) to supersymmetric particle jets evolution, we firstly review the way of treating ordinary particle jet evolution as branching process, with discussion about evolution parameter. In addition, we analyze the composition of Generalized Multiplicity Distribution (GMD) generating function, which will give hints for composition of multiplicity generating function for supersymmetric plus ordinary parton jets.

3.1 Branching Properties of QCD Jets

To discover the branching properties of QCD jets, we need to have a close look at Konishi *et al.* [18] and Giovannini [10] papers.

Based on the simple jet calculus algorithm, Konishi [18] derived two differential equations satisfied by multiplicity generating functions of single quark- and single gluon- initiated jets $P_q(x_q, x_g, Y)$, $P_g(x_q, x_g, Y)$ in the parton cascading in Leading Logarithmic Approximation (LLA).

$$
\begin{aligned}
P_q(x_q, x_g, Y) &= \sum_{n_q, n_g = 0}^{\infty} x_q^{n_q} x_g^{n_g} p(q \to n_{quarks} + n_{gluons}; Y) \\
P_g(x_q, x_g, Y) &= \sum_{n_q, n_g = 0}^{\infty} x_q^{n_q} x_g^{n_g} p(g \to n_{quarks} + n_{gluons}; Y)
\end{aligned}
\tag{3.1}
$$

The differential equation is valid for jets in semi-hard and hard interaction, with jet transverse momentum 1 GeV $\ll |p_T| \ll \sqrt{s}$.

$$
\begin{cases}
\dfrac{d}{dY} P_g & = A_0^{gg}(P_g^2 - P_g) + A_0^{qg}(P_q^2 - P_g) \\
\dfrac{d}{dY} P_q & = A_0^{gq}(P_g P_q - P_q)
\end{cases}
\tag{3.2}
$$

with initial condition as

$$
\begin{cases}
P_q(x_q, x_g, Y = 0) = x_q \\
P_g(x_q, x_g, Y = 0) = x_g
\end{cases}
\tag{3.3}
$$

In Giovannini's paper [10], he got the same differential equations for a Markov branching process involving the three basic processes :

(1) gluon splitting $g \to g + g$ with $A\Delta t$ as the probability for one gluon splitting into two in time interval Δt;

(2) quark bremsstrahlung $q \to q + g$ with $\tilde{A}\Delta t$ as probability for one quark emitting a gluon in time interval Δt;

(3) quark pair creation $g \to q + \bar{q}$ with $B\Delta t$ probability for one gluon splitting

into quark-antiquark pair in time interval Δt.

$$\begin{cases} \dfrac{\partial G}{\partial Y} & = A(G^2 - G) + B(Q^2 - G) \\ \dfrac{\partial Q}{\partial Y} & = \tilde{A}(QG - Q) \end{cases} \tag{3.4}$$

Here the generating functions for gluon and quark are G and Q. Comparing with equation 3.2 and equation 3.4, one can find the correspondence between the parameters of equation 3.2 and equation 3.4 : $A_0^{gg} = A$, $A_0^{gq} = \tilde{A}$, $A_0^{qg} = B$. That reveals the branching properties of QCD jets evolution.

The derivation of Konishi's elegant result [18] is based on the Leading Logarithmic Approximation(LLA) [8]. LLA means a kind of approximation for proton structure function $W(x, Q^2)$.

$$W(x, Q^2) \propto G_h^a(x, \ln Q^2) = \sum_{n=0}^{\infty} f_n(x)(\frac{\alpha_s}{\pi} \ln Q^2)^n \tag{3.5}$$

$G_h^a(x, \ln Q^2)$ represents the parton density function (equation 1.1). The Q^2 denotes the square of characterized momentum transfer in a hard process. The x denotes the fraction of longitudinal momentum carried by parton a from proton h. Only considering terms $(\frac{\alpha_s}{\pi} \ln Q^2)^n$ is the Leading Logarithmic Approximation. Also, in LLA of QCD, there is no interference terms for the tree diagram of parton showering, which gives the support for the probabilistic interpretation of parton showering.

The Y in the differential equation 3.2 is the "energy evolution parameter", defined as

$$Y = \frac{1}{2\pi b} \ln \frac{\alpha_s(Q_0^2)}{\alpha_s(Q^2)} = \frac{1}{2\pi b} \ln \frac{\ln(Q^2/\Lambda^2)}{\ln(Q_0^2/\Lambda^2)} \tag{3.6}$$

Here we use the one-loop expression for running coupling constant $\alpha_s(Q^2) = (b\ln(Q^2/\Lambda^2))^{-1}$ with $2\pi b = (11N_c - 2N_f)/6$, and the N_c, N_f represent the number of colors and flavors respectively. Running coupling constant means that the α_s depends on Q^2 (splitting scale). The expression of Y can be seen as "time parameter" for parton branching: The primary parton with virtuality [1] $t_{\max} = Q^2_{\max}$ ($Y = Y_{\max} = \dfrac{1}{2\pi b}\ln\dfrac{\ln(Q^2_{\max}/\Lambda^2)}{\ln(Q_0^2/\Lambda^2)}$) starts branching, by branching out new particles the virtuality of partons decreases, until the virtuality of the partons reaches $t_{min} = Q_0^2$ ($Y = 0$). The $t_{min} = Q_0^2$ is the non-perturbative threshold, partons will hadronize into hadrons for $t < Q_0^2$. One will notice the Y decreases from $Y_{\max}$ to $Y = 0$ through the branching, which is contradictory to the usual "time parameters". However, we will prove that the system which evolves from $Y_{\max}$ to $Y = 0$ is equivalent to system that evolves from $Y = 0$ to $Y_{\max}$ with the same initial condition, in next section of this chapter. What's more, Giovannini [10] proposed to interpret Y or $Y_{\max}$ as thickness of jets, i.e. the total time that the jet can evolve.

The branching probabilities A_0^{gg}, A_0^{qg} and A_0^{gq} in equation 3.2 are the zeroth moment of parton decay probability $P_{ba}(x)$, according to the expression of A_n^{ba}.

$$
\begin{aligned}
A_n^{ba} &= \int_0^1 dx x^n P(a \to b(x) + c(1-x)) \\
&= \int_0^1 dx x^n P_{ba}(x)
\end{aligned}
\tag{3.7}
$$

The parton decay probability function $P_{ba}(x)$ denotes the probability that parton a decays into parton b and other things (denoted by c), with b carrying fraction x of the longitudinal momentum of parton a. The parton decay probability functions [18] are listed in Table 3.1. Giovannini and Ugoccioni [12] proposed a

[1]The virtuality t measures how much a particle is off shell, with equation $t = p^\mu p_\mu - m^2$. Here $t = p_\perp^2 = Q^2$ is the square of transverse momentum.

Table 3.1 Parton decay probability functions

b	a	$P(a \to b(x) + c(1-x)) \equiv P_{ba}(x)$
q $\bar{q}$	q $\bar{q}$	$C_F(\frac{1+x^2}{1-x})_+$
g g	q $\bar{q}$	$C_F\frac{1+(1-x)^2}{x}$
q $\bar{q}$	g g	$\frac{1}{2}N_f[x^2+(1-x)^2]$
g	g	$2C_A[\frac{1-x}{x}+(\frac{x}{1-x})_+ + x(1-x) - \frac{1}{12}\delta(x-1)] - \frac{1}{3}N_f\delta(x-1)$

Here $C_F = (N_c^2-1)/2N_c, \quad C_A = N_c$ are color factors, and the integration rules for$(f(x))_+$: $\int dx(f(x))_+ g(x) = \int_0^1 dx f(x)[g(x)-g(1)]$

way to calculate the coefficients A_n^{ba}, which can resolve the infrared divergence problem. The way is to impose a fixed cut-off (small constant value $\epsilon' \ll 1$) on the fraction x of the longitudinal momentum carried by emitted parton b, i.e. $x_{min} = \epsilon'$ and $x_{\max} = 1-\epsilon'$.

Utilizing this method, we get the three coefficients:

$$
\begin{aligned}
A = A_0^{gg} &= \int_{\epsilon'}^{1-\epsilon'} P_{gg}(x)dx \\
&= 2C_A\int_{\epsilon'}^{1-\epsilon'}[\frac{1-x}{x}+x(1-x)]dx + 2C_A\int_{\epsilon'}^{1-\epsilon'}\frac{x}{1-x}(1-1)dx \\
&\quad -(\frac{C_A}{6}+\frac{N_f}{3})\int_{\epsilon'}^{1-\epsilon'}\delta(x-1) \\
&= 2C_A\int_{\epsilon'}^{1-\epsilon'}[\frac{1-x}{x}+x(1-x)] \\
&= C_A(-\frac{5}{3} - 2\ln\epsilon' + 2\epsilon' - 3\epsilon'^2 + O(\epsilon'^3))
\end{aligned}
\tag{3.8}
$$

$$
\begin{aligned}
\tilde{A} = A_0^{gq} &= \int_{\epsilon'}^{1-\epsilon'} P_{gq}(x)dx \\
&= C_F(-\frac{3}{2} - 2\ln\epsilon' + \epsilon' - \epsilon'^2 + O(\epsilon'^3))
\end{aligned}
\tag{3.9}
$$

Notice $\epsilon' \ll 1 \to \ln \epsilon' \gg 1 \gg \epsilon'$, we can omit $O(\epsilon)$, $O(1)$ and use the substitution $\epsilon = (-2\ln \epsilon')^{-1}$, notice $\epsilon \ll 1$

$$A \approx \frac{C_A}{\epsilon} = \frac{N_c}{\epsilon} \tag{3.10}$$

$$\tilde{A} \approx \frac{C_F}{\epsilon} = \frac{N_c^2 - 1}{2N_c\epsilon} \tag{3.11}$$

$$\begin{aligned} B = A_0^{qg} &= \int_{\epsilon'}^{1-\epsilon'} P_{qg}(x)dx \\ &= \frac{N_f}{2}(\frac{2}{3} - 2\epsilon' + 2\epsilon'^2 - \frac{4\epsilon'^3}{3}) \\ &\approx \frac{N_f}{3} \end{aligned} \tag{3.12}$$

3.2 Quark Initiated Jets and Gluon Initiated Jets

Giovannini [10] also proved the equivalence of equation 3.4 with the differential equation below.

$$\begin{aligned} \frac{dp_{n_g,n_q}(Y)}{dY} &= -An_g p_{n_g,n_q}(Y) + A(n_g - 1)p_{n_g-1,n_q}(Y) - \tilde{A}n_q p_{n_g,n_q}(Y) \\ &+ \tilde{A}n_q p_{n_g-1,n_q}(Y) - Bn_g p_{n_g,n_q}(Y) + B(n_g + 1)p_{n_g,n_q-2}(Y) \end{aligned} \tag{3.13}$$

The multiplicity distribution $p_{n_g,n_q}(Y)$ can be pure gluon initiated jets multiplicity distribution $p_{a,0;n_g,n_q}(Y)$ (number of initial gluons a), pure quark initiated jet multiplicity distribution $p_{0,d;n_g,n_q}(Y)$ (number of initial quarks d) or general multiplicity distribution $p_{n_g^0,n_q^0;n_g,n_q}(Y)$, with arbitrary initial number of gluons n_g^0 and quarks n_q^0. One can also include the $g \to g + g + g$ process with $C\Delta t$ as the infinitesimal probability. There are many attempts to solve this general differential equation to get an analytical expression for multiplicity distribution[6, 9].

To see the properties of the parton branching process in hadron collision, we start from the pure quark- or gluon- initiated jets. If we consider an approximation $B = 0$ ($A, \tilde{A} \gg B$ since $\epsilon \ll 1$), which means the gluon splitting into quark-aitiquark pairs can be neglected. In addition, we only consider the gluon numbers for multiplicity contribution $n = n_g$. The stochastic equations for multiplicity distribution of gluon- and quark- initiated jets read as:

$$\frac{dp_{a,0;n,0}(Y)}{dY} = -Anp_{a,0;n,0}(Y) + A(n-1)p_{a,0;n,0}(Y) \tag{3.14}$$

$$\begin{aligned} \frac{dp_{0,d;n,d}(Y)}{dY} =& - Anp_{0,d;n,d}(Y) + A(n-1)p_{0,d;n-1,d}(Y) \\ & - d\tilde{A}p_{0,d;n,d}(Y) + d\tilde{A}p_{0,d;n-1,d}(Y) \end{aligned} \tag{3.15}$$

The initial conditions for these two equations are:

$$\begin{aligned} p_{a,0;n,0}(0) &= 1 \\ p_{0,d;n,d}(0) &= 1 \end{aligned} \tag{3.16}$$

One should notice here the initial quarks or gluons should have the same virtuality $t_{max} = Q^2_{max}$ (same energy evolution parameter Y_{max}, or the same total evolution time for branching).

For pure gluon initiated jets, the branching is just the Simple Birth Process $A : g \to g + g$. The solution $p_{a,0;n,0}(Y)$ is standard Furry-Yule distribution

(FYD)

$$p_{a,0;n,0}(Y) = (e^{-AY})^a(1-e^{-AY})^{n-a}\frac{\Gamma(n)}{\Gamma(a)\Gamma(n-a+1)} \tag{3.17}$$

with its probability generating function as:

$$P_G(x,Y) = \left[\frac{x}{(1-e^{AY})x+e^{AY}}\right]^a \tag{3.18}$$

For pure quark initiated jets, the branching is the Simple Birth Process A : $g \to g+g$ plus Poisson Process $\tilde{A}$: $q \to q+g$. Here the quark number is constant $n_q = n_q^0 = d$ then $d\tilde{A}$ is just the parameter ν for Poisson Process. The m quarks can be seen as immigrants, which can give birth to ordinary individuals (gluons). The solution has been arrived from discussion of last chapter (equation 2.32 for phase one), with $\lambda = A$ and $\nu_1 = d\tilde{A}$. Here we introduce $k = d\tilde{A}/A$, the distribution is the Negative Binomial Distribution (NBD).

$$p_{0,d;n,d} = (1-e^{-AY})^n(e^{-AY})^k\frac{\Gamma(n+k)}{\Gamma(k)\Gamma(n+1)} \tag{3.19}$$

and its generating function

$$P_Q(x,Y) = \left[\frac{1}{(1-e^{AY})x+e^{AY}}\right]^k \tag{3.20}$$

3.2.1 Energy Evolution Parameter

In the former section 3.1 of this chapter, we introduce the energy evolution parameter Y. Here we will use the pure gluon initiated jets branching process to verify the equivalence of system evolving from $Y_{\max}$ to $Y = 0$ and of system evolving from $Y = 0$ to $Y_{\max}$, i.e. to prove the multiplicity distributions of these two system are the same.

The multiplicity distribution and generating function for the system evolving from $Y = 0$ to $Y_{\max}$ are equation 3.17 and equation 3.18. We just need to calculate for the system evolving from $Y_{\max}$ to $Y = 0$.

The initial condition is there are $n_g^0 = a$ initial gluons, and they all have the same virtuality $t_{\max} = Q_{max}^2$ (same $Y_{\max}$). Firstly let us find out the differential equation satisfied by multiplicity distribution $p_n(Y)$. Following the basic procedures, the probability of having n individuals at $Y - \Delta Y$ is

$$p_n(Y - \Delta Y) = p_{n-1}(Y) \cdot A(n-1)\Delta Y + p_n(t) \cdot (1 - An\Delta Y) \tag{3.21}$$

The differential equation can be arrived by taking ΔY to be infinitesimal.

$$-\frac{dp_n(Y)}{dY} = -\lim_{\Delta Y \to 0} \frac{p_n(Y) - p_n(Y - \Delta Y)}{\Delta Y} = A(n-1)p_{n-1}(Y) - Anp_n(Y) \tag{3.22}$$

then multiplying both side of the equation by x^n and summing over n, we get the equation for probability generating function $P(x, Y) = \sum_{n=0}^{\infty} p_n(Y)x^n$.

$$-\frac{\partial P(x,Y)}{\partial Y} = Ax^2\frac{\partial P(x,Y)}{\partial x} - Ax\frac{\partial P(x,Y)}{\partial x} \tag{3.23}$$

for moment generating function, we substitute $x = e^{\theta}$ and get:

$$-\frac{\partial M}{\partial Y} = A(e^{\theta} - 1)\frac{\partial M}{\partial \theta} \tag{3.24}$$

The subsidiary equations are :

$$-\frac{dY}{1} = \frac{d\theta}{-A(e^{\theta} - 1)} = \frac{dM}{0} \tag{3.25}$$

The first and last term give:

$$M = const. \tag{3.26}$$

while the first and second term result in:

$$-dY = \frac{e^{-\theta} d\theta}{-A(1 - e^{-\theta})} \tag{3.27}$$

Integrate the equation above from $Y_{\max}$ to Y

$$\begin{aligned} -(Y - Y_{\max}) &= \frac{\ln(1 - e^{-\theta})}{-A} - const. \\ e^{A(Y_{\max} - Y)}(1 - e^{-\theta}) &= const. \end{aligned} \tag{3.28}$$

The general solution of the differential equation 3.24 can be written as:

$$M(\theta, Y) = \Psi[e^{A(Y_{\max} - Y)}(1 - e^{-\theta})] \tag{3.29}$$

with initial condition being

$$M(\theta, Y_{\max}) = e^{a\theta} \tag{3.30}$$

the general solution at $Y = Y_{\max}$ reads

$$e^{a\theta} = \Psi[(1 - e^{-\theta})] \tag{3.31}$$

using substitution

$$u = (1 - e^{-\theta}) \rightarrow e^{\theta} = \frac{1}{1-u} \tag{3.32}$$

we find

$$\Psi[u] = \left(\frac{1}{1-u}\right)^{a} \tag{3.33}$$

The general solution is

$$M(\theta, Y) = \Psi[e^{A(Y_{\max}-Y)}(1 - e^{-\theta})] = \left[\frac{1}{1 - e^{A(Y_{\max}-Y)}(1 - e^{-\theta})}\right]^{a} \tag{3.34}$$

At $Y = 0$, the momentum generating function reads

$$M(\theta, 0) = \left[\frac{1}{1 - (1 - e^{-\theta})e^{AY_{\max}}}\right]^{a} \tag{3.35}$$

multiplicity generating function reads

$$P(x, 0) = \left[\frac{1}{1 - (1 - x^{-1})e^{AY_{\max}}}\right]^{a} = \left[\frac{x}{(1 - e^{AY_{\max}})x + e^{AY_{\max}}}\right]^{a} \tag{3.36}$$

multiplicity distribution reads

$$p(0) = (e^{-AY_{\max}})^{a}(1 - e^{-AY_{\max}})^{n-a}\frac{\Gamma(n)}{\Gamma(a)\Gamma(n-a+1)} \tag{3.37}$$

Comparing with equation 3.18 and equation 3.17, they are exactly the same probability generating function and multiplicity distribution. So later on, when we encounter the system which evolves from $Y_{\max}$ to $Y = 0$, we can just solve the system evolving from $Y = 0$ to $Y_{\max}$ with same initial condition.

3.3 Generalized Multiplicity Distribution

Chew *et al.* [6] gave a multiplicity distribution for arbitrary initial number of quarks d and gluons a, which is the solution to equation 3.13, the general differential equation for parton branching under approximation $B = 0$, with $k = d\tilde{A}/A$:

$$p_{\text{GMD}}(Y) = e^{-AY(k+a)}[1 - e^{-AY}]^{n-a}\frac{\Gamma(n+k)}{\Gamma(n-a+1)\Gamma(k+a)} \tag{3.38}$$

it is called Generalized Multiplicity Distribution (GMD), its generating function

$$P_{\text{GMD}}(x, Y) = \frac{x^a}{[(1 - e^{AY})x + e^{AY}]^{k+a}} \tag{3.39}$$

Based on our understanding about the energy evolution parameter Y, we know the expression for GMD implies the fact that all the initial partons should have same virtuality $t_{\max}$ or $Y_{\max}$, and they branch out independently. To see this point more clearly, one can rewrite the GMD generating function as:

$$\begin{aligned} P_{\text{GMD}}(x, Y_{\max}) &= P_G(x, Y_{\max}) \times P_Q(x, Y_{\max}) \\ &= \left[\frac{x}{(1 - e^{AY_{\max}})x + e^{AY_{\max}}}\right]^a \times \left[\frac{1}{(1 - e^{AY_{\max}})x + e^{AY_{\max}}}\right]^k \end{aligned} \tag{3.40}$$

From the expression, the GMD generating function is the convolution of gluon initiated generating function (equation 3.18) and quark initiated generating function (equation 3.20), with same $Y_{\max}$. The initial quarks and gluons can be seen as independent sources individually.

The physical meaning behind GMD lies in the stochastic branching equations which are originated from perturbative QCD calculation (Konishi jet calculus algorithm [18]), it is also the common properties shared by all the distributions derived from stochastic branching equations. The GMD and stochastic branching equations are only for semi-hard and hard jets, i.e. energy range $1 \text{ GeV} \ll |p_T| \ll \sqrt{s}$ [18], since pQCD is only valid for this region. In the hadronic collisions till now, the dominant contribution to multiplicity distribution is from soft interaction. One assumes the branching of partons from soft interaction also follows the GMD, and get good fitting result for hadronic collisions up to $\sqrt{s} = 900$ GeV [4]. For hadronic collisions with $\sqrt{s} > 900$ GeV, one can get good fitting result with two component GMD [7]. These are the same as well-known Negative Binomial Distribution.

Chapter 4

SUSY-QCD Jets Branching Process

Supersymmetry (SUSY), as one of the hottest topics of the beyond standard model physics, attracts great interests. There are various theoretical supersymmetry models, and different experimental searches for supersymmetric particles, such as the CMS-SUSY group and ATLAS-SUSY group at LHC. However, the discussion of supersymmetry particle jet evolution from phenomenological point of view is few, here we would like to describe the evolution of supersymmetric particle jets based on our modified stochastic branching model.

This chapter will start by brief introduction about supersymmetry, then come to the supersymmetric particle jets branching process discussion, and finally to the general case – supersymmetric plus ordinary parton jets branching process discussion.

4.1 Supersymmetry

Supersymmetry is a new symmetry between fermions and bosons. It assumes all existing ordinary elementary particles have its superparterners (supersymmetric particles). The ordinary particles and superparterners differ in spin by half. The supersymmetry is spontaneously broken, which results in the mass difference between superparterners and normal particles. The superparterners are much heavier than ordinary particles, and haven't been found in Large Hadron Collider (LHC) till now. Another important property of supersymmetry theory is the R-Parity. The R-Parity of a particle is defined as $R_P = (-1)^{3(B-L)+2s}$, with B for baryon number, L for lepton number and s for spin. SUSY particle has R-parity -1, while ordinary particle R-parity 1. Some Supersymmetry models assume that R-Parity conserves, some do not. If R-Parity conserves (simpler case), any process should involve even number of SUSY particles. This has direct phenomenological consequence: there exists the Lightest Supersymmetryic Particle (LSP), it is stable and can not decay. If it decays into ordinary particles, this decay process only involves one (odd) number of particles.

The motivations of supersymmetry come from several aspects [14]. First one is the quadratic Higgs mass divergence, adding SUSY particles can cancel the coefficient of divergence term in Higgs boson mass expression. The second is that the spin degree of freedom has not been taken into account in the present gauge theory, but in supersymmetry theory. The third is that locally supersymmetric theory relates supersymmetric transformation with spacetime transformation, which provides opportunity to unify gravity with strong and electroweak force. What's more, the lightest supersymmetric particle (LSP) is a promising candidate for dark matter, since LSP is stable and weakly interact with other particles.

4.2 SUSY Particle Branching

Nowadays the SUSY particle searches concentrate on the on-shell decays of SUSY particles, since the energy of collision is not large enough to create off-shell SUSY particles to participate SUSY-QCD branching [3] (The data fitting in next section will show this point). There are Monte Carlo simulations [3] which include the SUSY particles branching. Here we try to construct the stochastic equation for pure SUSY particles branching process, and get the generating function and multiplicity distribution from it.

SUSY particles (partons) start branching with same high primary virtuality $t_{\max}$ (same $Y_{\max}$), they branch out particles and the virtuality decreases. Until the virtuality reaches the mass scale of SUSY partons $t = M^2_{\text{SUSY}}$ [1] ($Y = Y_{\text{SUSY}}$), the SUSY partons decay into ordinary partons and Lightest Supersymmetric Particles (LSP). The LSP will not be detected by detectors and the ordinary partons continue branching until their virtuality reach $t = Q_0^2$, i.e. $Y = 0$, the non-perturbative QCD threshold, then they will hadronize into hadrons. One will notice the system undergoes two phase branching process, the first phase is SUSY partons branching, the second is ordinary partons (generated by SUSY partons) branching. This two phase branching can be described by our modified branching model (Chapter 2), as to be discussed in the following.

All the specific SUSY parton branching processes and their decay probability functions $P_{ba}(x)$ [16] are listed in Table 4.1, with the corresponding SUSY parton branching probabilities A_0^{ba} calculated and listed in the last column.

Here we demonstrate the calculation of first two parton branching probabilities $A_0^{\tilde{q}q}$ and $A_0^{\tilde{q}\tilde{q}}$ as examples. The infrared cut-off we use is the same as for the ordinary parton, proposed by Giovannini and Ugoccioni [12] and discussed

[1] Here we suppose the squarks and gluinos are of the same mass M_{SUSY}

Table 4.1 SUSY Parton decay probability functions

b	a	$P(a \to b(x) + c(1-x)) \equiv P_{ba}(x)$	$A_0^{ba} = \int_{\epsilon'}^{1-\epsilon'} P_{ba}(x)dx$
$\tilde{q}$	q	$C_F x$	$\frac{C_F}{2}$
$\tilde{g}$	q	$C_F(1-x)$	
$\tilde{q}$	$\tilde{q}$	$C_F[\frac{2x}{1-x} - \delta(1-x)[1 + \int_0^1 \frac{2y}{1-y} dy]]$	$\frac{C_F}{\epsilon}$
g	$\tilde{q}$	$C_F \frac{2(1-x)}{x}$	
q	$\tilde{q}$	C_F	C_F
g	$\tilde{q}$		
$\tilde{q}$	g	$2N_f x(1-x)$	$\frac{N_f}{3}$
$\tilde{q}$	g		
$\tilde{g}$	g	$C_A[(1-y)^2 + y^2]$	$\frac{2C_A}{3}$
$\tilde{g}$	g		
$\tilde{g}$	$\tilde{g}$	$C_A[\frac{1+x^2}{1-x} - \delta(1-x)[\int_0^1 \frac{1+y^2}{1-y} dy + \frac{N_f}{6}]]$	$\frac{C_A}{\epsilon}$
g	$\tilde{g}$	$C_A \frac{1+(1-x)^2}{x}$	
$\tilde{q}$	$\tilde{g}$	$N_f x$	$\frac{N_f}{2}$
q	$\tilde{g}$	$N_f(1-x)$	

For SUSY-QCD, the color factor relation is $C_F = C_A = N_c$. Here the supersymmetric quark – squark is denoted by $\tilde{q}$, while supersymmetric gluon – gluino is $\tilde{g}$.

in Chapter 3.1. Instead of integrating from 0 to 1 for x, fraction of momentum carried by emitted parton, we integrate from $x_{min} = \epsilon'$ to $x_{max} = 1 - \epsilon'$. Notice that $\epsilon' \ll 1$ and we use substitution $\epsilon = (-2\ln\epsilon')^{-1}$, also $\epsilon \ll 1$.

$$\begin{aligned} A_0^{\tilde{q}q} &= C_F \int_{\epsilon'}^{1-\epsilon'} x dx \\ &\approx \frac{C_F}{2} \end{aligned} \tag{4.1}$$

$$\begin{aligned} A_0^{\tilde{q}\tilde{q}} &= C_F \int_{\epsilon'}^{1-\epsilon'} \frac{2x}{1-x} dx - C_F \int_{\epsilon'}^{1-\epsilon'} dx \delta(1-x)[1 + \int_0^1 \frac{2y}{1-y} dy] \\ &= C_F \int_{\epsilon'}^{1-\epsilon'} \frac{2x}{1-x} dx \\ &= 2C_F(-1 - \ln\epsilon' + \epsilon' - \frac{\epsilon'^2}{2} + O(\epsilon'^3)) \\ &\approx \frac{C_F}{\epsilon} \end{aligned} \tag{4.2}$$

In SUSY-QCD, the color factor relation becomes $C_F = C_A = N_c$, since in SUSY theory, “quark” and “gluon” belong to the same representation of color group [8]. This changes the value of energy evolution parameter Y. Here we take the approximation of neglecting all the A_0^{ba} which do not contain $\frac{1}{\epsilon}$ since $\epsilon \ll 1$, similar to approximation $B = 0$ for ordinary parton branching process. In this case, only $\tilde{q} \to \tilde{q} + g$ and $\tilde{g} \to \tilde{g} + g$ two SUSY branching processes are left, accompanied by ordinary parton branching processes $q \to q + g$ and $g \to g + g$. All the possible parton branching probabilities are denoted as $A_0^{\tilde{q}\tilde{q}} \equiv I$, $A_0^{\tilde{g}\tilde{g}} \equiv J$, $A = A_0^{gg}$ and $\tilde{A} = A_0^{gq}$. One will notice this identity:

$$I = \frac{C_F}{\epsilon} = J = \frac{C_A}{\epsilon} = A = \frac{C_A}{\epsilon} = \tilde{A} = \frac{C_F}{\epsilon} \tag{4.3}$$

due to the color factor relation $C_F = C_A = N_c$ in SUSY-QCD. When we only consider the gluon contribution for multiplicity, the $\tilde{q} \to \tilde{q} + g$, $\tilde{g} \to \tilde{g} + g$, $q \to q + g$ can be seen as Poisson Process, with $\tilde{q}$ $\tilde{g}$, q as immigrants[2].

For pure SUSY particle initiated branching, the initial condition is squark initial number b, gluino initial number c and no quarks or gluons, i.e. only two kinds of immigrants squark $\tilde{q}$ and gluino $\tilde{g}$. This stochastic branching process consists of the Simple Birth Process $g \to g + g$ and Poisson Process $\tilde{q} \to \tilde{q} + g$, $\tilde{g} \to \tilde{g} + g$, with its stochastic equation as:

$$\frac{dp_n}{dY} = -Anp_n + A(n-1)p_{n-1} - (bI + cJ)p_n + (bI + cJ)p_{n-1} \tag{4.4}$$

using the substitutions

$$\lambda = A \tag{4.5}$$

$$v_1 = bI + cJ \tag{4.6}$$

the equation becomes

$$\frac{dp_n}{dY} = -\lambda np_n + \lambda(n-1)p_{n-1} - v_1 p_n + v_1 p_{n-1} \tag{4.7}$$

This equation is just the standard stochastic equation (equation 2.17) for Simple Birth Process plus Poisson Process. From $Y = 0$ to $Y = Y_1 = Y_{\max} - Y_{\mathrm{SUSY}}$, the branching process is described by this equation. At $Y_1 = Y_{\max} - Y_{\mathrm{SUSY}}$ (the virtuality of SUSY particles reaches M^2_{SUSY}), SUSY particles decay into ordinary particles and lightest supersymmetric particles (LSP) $\tilde{\chi}^0_1$. Here we consider the

[2]Refer to explanation in Chapter 2 about immigrant in introduction about Poisson Process

direct decay of $\tilde{q}$ and $\tilde{g}$ [3].

$$\tilde{q} \to q + \tilde{\chi}_1^0 \tag{4.8}$$

$$\tilde{g} \to q + \bar{q} + \tilde{\chi}_1^0 \tag{4.9}$$

The b squarks become b ordinary quarks, and c gluinos become $2c$ ordinary quarks. The ordinary partons continue to branching, until $Y = Y_{max}$. Notice here the actual evolution of Y is the inverse direction, from $Y = Y_{max}$ to Y_1 and to $Y = 0$ (discussed in Chapter 3.2). The stochastic equation for phase two is as follows, now the quark number is $b + 2c$.

$$\frac{dp_n}{dY} = -Anp_n + A(n-1)p_{n-1} - (b+2c)\tilde{A}p_n + (b+2c)\tilde{A}p_{n-1} \tag{4.10}$$

using the substitutions

$$\lambda = A \tag{4.11}$$

$$\nu_2 = (b+2c)\tilde{A} \tag{4.12}$$

the equation becomes

$$\frac{dp_n}{dY} = -\lambda np_n + \lambda(n-1)p_{n-1} - \nu_2 p_n + \nu_2 p_{n-1} \tag{4.13}$$

These two stochastic equations 4.7 and 4.13 are just the equations for the two phases of our modified branching model, discussed in Chapter 2.2. The Simple Birth Process parameter λ stays unchanged, while the Poisson Process parameter is different for phase one ν_1 and phase ν_2 due to change of immigrants type

[3] This dacay channel is the simplest on-shell decay channel of the SUSY searches in CMS and ATLAS SUSY group.[22]

$(\tilde{q}, \tilde{g} \rightarrow q)$ and number. The probability generating function $P_{\mathrm{SUSY}}(x, Y)$ and multiplicity distribution $p_{\mathrm{SUSY}}(n, Y)$ are already derived in Chapter 2 ($k_1 = \nu_1/\lambda$, $k_2 = \nu_2/\lambda$):

$$P_{\mathrm{SUSY}}(x, Y) = \frac{[(1 - e^{\lambda(Y-Y_1)})x + e^{\lambda(Y-Y_1)}]^{k_1-k_2}}{[(1 - e^{\lambda Y})x + e^{\lambda Y}]^{k_1}} \tag{4.14}$$

$$\begin{aligned} p_{\mathrm{SUSY}}(n, Y) = \quad & e^{\lambda(Y-Y_1)(k_1-k_2)}(1 - e^{-\lambda Y})^n (e^{-\lambda Y})^{k_1} \frac{\Gamma(n + k_1)}{\Gamma(k_1)\Gamma(n + 1)} \\ & \times {}_2F_1(k_2 - k_1, -n, 1 - k_1 - n, \frac{e^{\lambda Y} - e^{\lambda Y_1}}{e^{\lambda Y} - 1}) \end{aligned} \tag{4.15}$$

4.3 SUSY plus Ordinary Parton Branching

From the discussion in Chapter 3.2.2, we know that Generalized Multiplicity Distribution (GMD) generating function for arbitrary initial number of quarks and gluons (with the same primary virtuality t_{max} and same Y_{max}) is just the convolution of multiplicity generating function for quark initiated distribution and that for gluon initiated distribution, shown in 3.40. This is due to the independence of jets. Following the same consideration, we can get the multiplicity generating function $P_{\mathrm{mix}}(x)$ for b (initial particle number, same as c, d, a) squarks, c gluinos, d quarks and a gluons of same primary virtuality t_{max} (same Y_{max}).

The stochastic equations for b squarks, c gluinos, d quarks initiated jets (only immigrants at the beginning) of two phase $[0, Y_1]$ $[Y_1, Y_{max}]$ are

$$\begin{aligned} \frac{dp_n}{dY} &= -Anp_n + A(n-1)p_{n-1} - (d\tilde{A} + bI + cJ)p_n + (d\tilde{A} + bI + cJ)p_{n-1} \\ \frac{dp_n}{dY} &= -Anp_n + A(n-1)p_{n-1} - (d + b + 2c)\tilde{A}p_n + (d + b + 2c)\tilde{A}p_{n-1} \end{aligned} \tag{4.16}$$

One can find the equations are the same as pure SUSY initiated jets stochastic equations, only with different ν_1 and ν_2 substitution

$$\begin{aligned} \nu_1 &= (d\tilde{A} + bI + cJ) \\ \nu_2 &= (d + b + 2c)\tilde{A} \end{aligned} \tag{4.17}$$

The generating function $P_{\tilde{Q}\tilde{G}Q}(x,t)$ is as follow, with $k_1 = \nu_1/\lambda$, $k_2 = \nu_2/\lambda$

$$P_{\tilde{Q}\tilde{G}Q}(x,Y) = \frac{[(1-e^{\lambda(Y-Y_1)})x + e^{\lambda(Y-Y_1)}]^{k_1-k_2}}{[(1-e^{\lambda Y})x + e^{\lambda Y}]^{k_1}} \tag{4.18}$$

The stochastic equation for a gluons initiated jets (pure gluon bremsstrahlung) is the same for two phase $[0,Y_1]$ $[Y_1,Y_{max}]$:

$$\frac{dp_n}{dY} = -Anp_n + A(n-1)p_{n-1} \tag{4.19}$$

The generating function $P_G(x,Y)$ is as follow, notice $\lambda = A$.

$$P_G(x,Y) = \left[\frac{x}{(1-e^{\lambda Y})x + e^{\lambda Y}}\right]^a \tag{4.20}$$

The generating function for mixed initial partons (squarks, gluinos, quarks and gluons) $P_{\text{mix}}(x)$ is just the convolution of $P_{\tilde{Q}\tilde{G}Q}$ and $P_G(x,Y)$.

$$\begin{aligned} P_{\text{mix}}(x,Y) &= P_{\tilde{Q}\tilde{G}Q}(x,Y) \times P_G(x,Y) \\ &= \frac{[(1-e^{\lambda(Y-Y_1)})x + e^{\lambda(Y-Y_1)}]^{k_1-k_2}}{[(1-e^{\lambda Y})x + e^{\lambda Y}]^{k_1}} \times \left[\frac{x}{(1-e^{\lambda Y})x + e^{\lambda Y}}\right]^a \end{aligned} \tag{4.21}$$

We get the multiplicity distribution $p(n, Y)$ by expanding $P_{\text{mix}}(x, Y)$ in Taylor series. Firstly rearrange the expression as

$$\begin{aligned}
P_{\text{mix}}(x, Y) &= [(1 - e^{\lambda(Y-Y_1)})x + e^{\lambda(Y-Y_1)}]^{k_1-k_2} \frac{x^a}{[(1 - e^{\lambda Y})x + e^{\lambda Y}]^{k_1+a}} \\
&= e^{\lambda(Y-Y_1)(k_1-k_2)}[1 - (1 - e^{-\lambda(Y-Y_1)})x]^{k_1-k_2} \frac{(e^{\lambda Y})^a (e^{-\lambda Y} x)^a}{(e^{\lambda Y})^{k_1+a}[1 - (1 - e^{-\lambda Y})x]^{k_1+a}}
\end{aligned} \tag{4.22}$$

then introduce substitutions:

$$\begin{aligned}
e^{-\lambda Y} &= p \\
1 - e^{-\lambda Y} &= q \\
e^{-\lambda(Y-Y_1)} &= r \\
1 - e^{-\lambda(Y-Y_1)} &= s
\end{aligned} \tag{4.23}$$

the generating function can be rewritten as

$$\begin{aligned}
P_{\text{mix}}(x, Y) &= (\frac{1}{r})^{k_1-k_2}[1 - sx]^{k_1-k_2} \frac{(px)^a}{(\frac{1}{p})^{k_1}[1 - qx]^{k_1+a}} \\
&= r^{k_2-k_1} p^{k_1} (px)^a \sum_{m=0}^{\infty} \frac{(m + k_2 - k - 1 - 1)!}{m!\,(k_2 - k_1 - 1)!}(sx)^m \sum_{l=0}^{\infty} \frac{(l + k_1 + a - 1)!}{l!\,(k_1 + a - 1)!}(qx)^l \\
&= r^{k_2-k_1} p^{k_1+a} x^a \sum_{n=0}^{\infty} \sum_{m=0}^{n} \frac{(m + k_2 - k_1 - 1)!}{m!\,(k_2 - k_1 - 1)!}(sx)^m \frac{(n - m + k_1 + a - 1)!}{(n - m)!\,(k_1 + a - 1)!}(qx)^{n-m} \\
&= r^{k_2-k_1} p^{k_1+a} \sum_{n=0}^{\infty} q^n \frac{(n + k_1 + a - 1)!}{(k_1 + a - 1)!\,n!} {}_2F_1[k_2 - k_1, -n, 1 - k_1 - a - n, \frac{s}{q}] x^{n+a}
\end{aligned} \tag{4.24}$$

From the expansion above, we know the multiplicity of p_{n+a}

$$p_{n+a} = r^{k_2-k_1} p^{k_1+a} q^n \frac{\Gamma(n+k_1+a)}{\Gamma(k_1+a)\Gamma(n+1)} {}_2F_1[k_2-k_1, -n, 1-k_1-a-n, \frac{s}{q}] \tag{4.25}$$

the SUSY-included multiplicity distribution (SIMD) $p_{\mathrm{SIMD}}(n,Y)$ is arrived by substitution $n' = n + a$.

$$\begin{aligned} p_{\mathrm{SIMD}}(n,Y) &= r^{k_2-k_1} p^{k_1+a} q^{n-a} \frac{\Gamma(n+k_1)}{\Gamma(k_1+a)\Gamma(n-a+1)} {}_2F_1[k_2-k_1, -n+a, 1-k_1-n, \frac{s}{q}] \\ &= e^{\lambda(Y-Y_1)(k_1-k_2)} (e^{-\lambda Y})^{k_1+a} (1-e^{-\lambda Y})^{n-a} \frac{\Gamma(n+k_1)}{\Gamma(k_1+a)\Gamma(n-a+1)} \\ &\quad \times {}_2F_1[k_2-k_1, -n+a, 1-k_1-n, \frac{e^{\lambda Y}-e^{\lambda Y_1}}{e^{\lambda Y}-1}] \end{aligned} \tag{4.26}$$

with substitutions equation 4.23 to replace p, q, r, s.

4.3.1 Generating Function and Multiplicity Distribution

The generating function $P_{\mathrm{mix}}(x,Y)$ and multiplicity distribution $p_{\mathrm{SIMD}}(n,Y)$ for arbitrary initial number of SUSY and ordinary partons are as follows.

$$P_{\mathrm{mix}}(x,Y) = \frac{[(1-e^{\lambda(Y-Y_1)})x + e^{\lambda(Y-Y_1)}]^{k_1-k_2}}{[(1-e^{\lambda Y})x + e^{\lambda Y}]^{k_1}} \times \left[\frac{x}{(1-e^{\lambda Y})x + e^{\lambda Y}}\right]^a \tag{4.27}$$

$$\begin{aligned} p_{\mathrm{SIMD}}(n,Y) =& e^{\lambda(Y-Y_1)(k_1-k_2)} (e^{-\lambda Y})^{k_1+a} (1-e^{-\lambda Y})^{n-a} \frac{\Gamma(n+k_1)}{\Gamma(k_1+a)\Gamma(n-a+1)} \\ & \times {}_2F_1[k_2-k_1, -n+a, 1-k_1-n, \frac{e^{\lambda Y}-e^{\lambda Y_1}}{e^{\lambda Y}-1}] \end{aligned} \tag{4.28}$$

Here $\lambda = A$, A is the branching probability for $g \to g + g$. Also

$$\begin{aligned} k_1 &= \frac{\nu_1}{\lambda} = \frac{d\tilde{A} + bI + cJ}{A} \\ k_2 &= \frac{\nu_2}{\lambda} = \frac{(d + b + 2c)\tilde{A}}{A} \end{aligned} \tag{4.29}$$

The I denotes the branching probability for $\tilde{q} \to \tilde{q} + g$, J for $\tilde{g} \to \tilde{g} + g$, $\tilde{A}$ for $q \to q + g$. The d is the initial number of quarks, while initial nuber of squarks, gluinos and gluons are b, c and a. The $Y_1 = Y_{\max} - Y_{\text{SUSY}}$, the $Y_{\max}$ represents the evolution parameter for max virtuality $t_{\max}$(the virtuality of primary partons), Y_{SUSY} denotes when the virtuality reaches SUSY particle mass scale M^2_{SUSY}.

One can do several consistency checks for the SUSY-included multiplicity distribution and its generating function:

1. When initial gluon number $a = 0$, this distribution (generating function) is reduced to pure SUSY particles initiated jets multiplicity distribution (generating function), equation 4.15 (equation 4.14).

2. When $Y_1 = Y$ or $k_1 = k_2 = k$, i.e. no SUSY particles involved, this distribution (generating function) is reduced to Generalized Multiplicity Distribution (generating function), equation 3.38 (equation 3.39).

4.3.2 Data Fitting

After deriving the SUSY-included multiplicity distribution, we fit the distribution with experimental data. For data fitting, we use some substitutions

$$\begin{aligned} m &= e^{\lambda Y_1} \\ l &= e^{\lambda Y} \end{aligned} \tag{4.30}$$

to simplify the distribution

$$\begin{aligned} p_{\mathrm{SIMD}}(n,Y) = \quad & (\frac{l}{m})^{k_1-k_2}(1-\frac{1}{l})^{n-a}(\frac{1}{l})^{k_1}\frac{(n+k_1-1)!}{(k_1+a-1)!\,(n-a)!} \\ & \times {}_2F_1(k_2-k_1,-n+a,1-k_1-n,\frac{l-m}{l-1}) \end{aligned} \tag{4.31}$$

The evolution parameter ordering $0 < Y_1 < Y$ will give us the constraint $1 < m < l$. For k_1 and k_2, when we use the identity 4.3, they will become:

$$\begin{aligned} k_1 &= \frac{\nu_1}{\lambda} = \frac{d\tilde{A} + bI + cJ}{A} = d + b + c \\ k_2 &= \frac{\nu_2}{\lambda} = \frac{(d+b+2c)\tilde{A}}{A} = d + b + 2c \end{aligned} \tag{4.32}$$

The c is the initial gluino number and $c > 0$, so we have another constraint $k_2 > k_1$ for data fitting.

Our SUSY-included multiplicity distribution $p_{\mathrm{SIMD}}(n,Y)$ is fitted with CMS data of pp collision at center of mass energy $\sqrt{s} = 0.9, 2.36, 7$ TeV for pseudorapidity intervals $|\eta| < 0.5, 1.0, 1.5, 2.0$ and 2.4. The best fitted parameters and χ^2/dof are tabulated in Table 4.2. We use CERN-MINUIT package to do the data fitting, under ROOT environment. As for the Hypergeometric function contained in the multiplicity distribution, we call the Mathematica for its evaluation. We attach our C code and fitting procedures in Appendix.

From the parameter values, for all the collision energy $\sqrt{s}$ and pseudorapidity range $|\eta|$ listed here, $k_1 = k_2$ or $m = l$ $(Y_1 = Y)$ is satisfied. That means the SUSY-included multiplicity distribution is reduced to Generalized Multiplicity Distribution (GMD) for current experimental data, and no SUSY particle participates branching. This fact is consistent with nowadays SUSY particle searches, with no signal of SUSY particle appeared for current collision energy. As mentioned by Monte Carlo paper from Berezinsky and Kachelriess [3], the current

Table 4.2 SUSY-included Multiplicity Distribution best fit parameters and χ^2/dof for CMS $\sqrt{s} = 0.9, 2.36, 7$ TeV data at different pseudorapidity intervals $|\eta| < 0.5, 1.0, 1.5, 2.0, 2.4$.

$\sqrt{s}$(TeV)	η_c	m	l	k_1	k_2	a	χ^2/dof(dof)
	0.5	3.25	3.25	1.61	1.61	0.102	0.106(17)
	1.0	4.84	4.84	2.07	2.07	8.88×10^{-3}	1.04(34)
0.9	1.5	6.77	6.77	2.05	2.05	1.74×10^{-8}	0.768(51)
	2.0	8.65	8.68	2.04	2.04	2.73×10^{-3}	0.526(54)
	2.4	9.65	9.83	2.13	2.13	3.59×10^{-4}	0.700(62)
	0.5	1.24	4.26	1.42	1.42	0.0746	0.338(17)
	1.0	1.10	6.29	1.90	1.90	5.47×10^{-12}	1.58(34)
2.36	1.5	1.57	9.75	1.70	1.70	4.72×10^{-3}	0.555(44)
	2.0	2.51	12.7	1.69	1.69	5.86×10^{-3}	0.603(54)
	2.4	3.21	14.4	1.77	1.77	3.98×10^{-3}	0.624(64)
	0.5	5.37	5.37	1.57	1.57	6.52×10^{-9}	2.05(35)
	1.0	9.58	9.58	1.54	1.72	3.44×10^{-11}	2.16(64)
7	1.5	13.48	13.48	1.58	1.89	6.60×10^{-10}	2.53(89)
	2.0	17.69	18.1	1.55	1.55	3.61×10^{-13}	1.97(109)
	2.4	22.5	22.5	1.48	1.60	0	1.08(121)

collision energy can only produce on-shell SUSY particles, while SUSY particle branching involves highly off-shell SUSY particles. The SUSY particle branching may happen in ultra-high energy cosmic ray. When data analyses on multiplicity distribution for ultra-high energy cosmic ray are available, one can apply this SUSY particle branching model to it.

We plot multiplicity data with our best fitted SUSY-included multiplicity distribution (SIMD) at $\sqrt{s} = 0.9$ TeV in Fig 4.1, $\sqrt{s} = 2.36$ TeV in Fig 4.2, and $\sqrt{s} = 7$ TeV in Fig 4.3 for different pseudorapidity intervals. Since the SUSY-included multiplicity distribution here is reduced to GMD, the figures here show the fitting quality of GMD, where the shoulder structure remains the main difficulty for multiplicity distribution now.

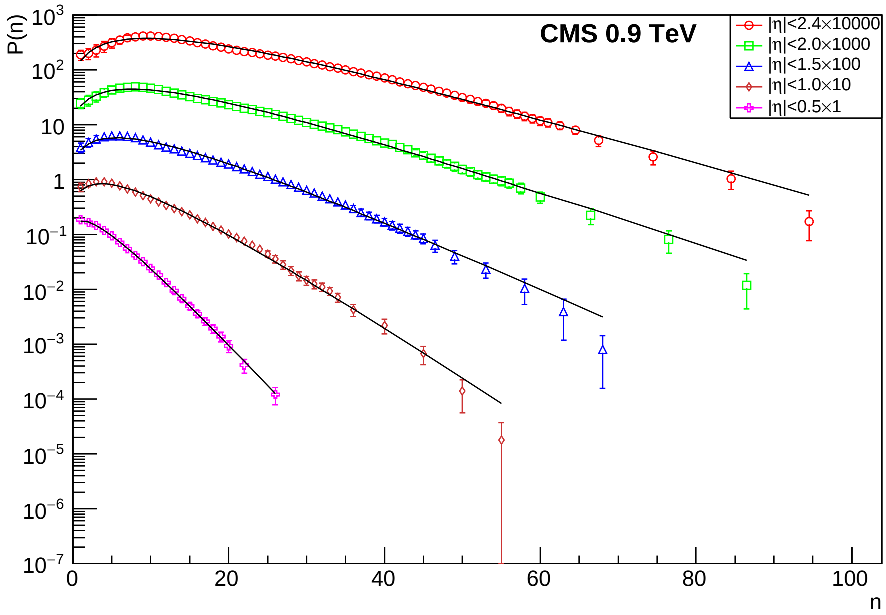

Fig. 4.1 Charged particle multiplicity distribution measured by CMS at $\sqrt{s}$ = 0.9TeV, fitted with SUSY-included model.

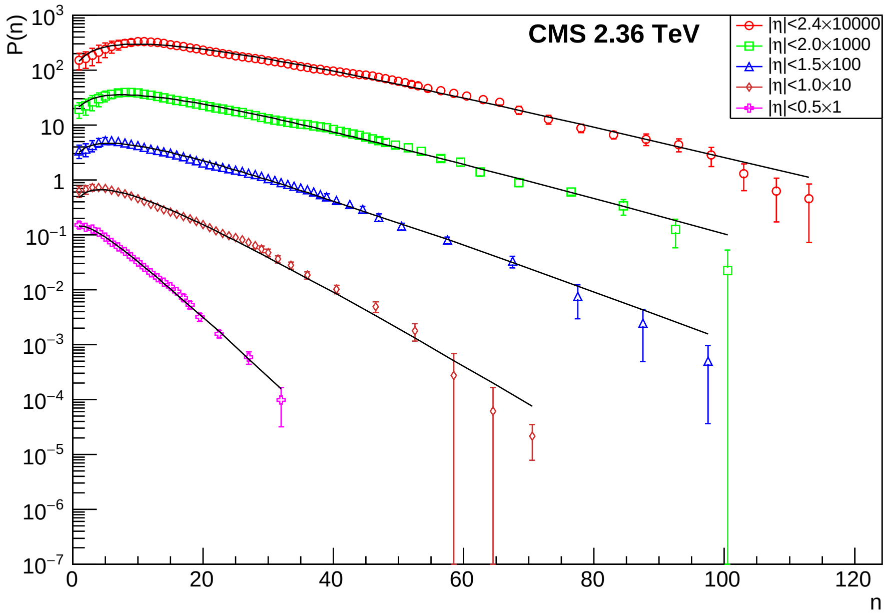

Fig. 4.2 Charged particle multiplicity distribution measured by CMS at $\sqrt{s}$ = 2.36TeV, fitted with SUSY-included model.

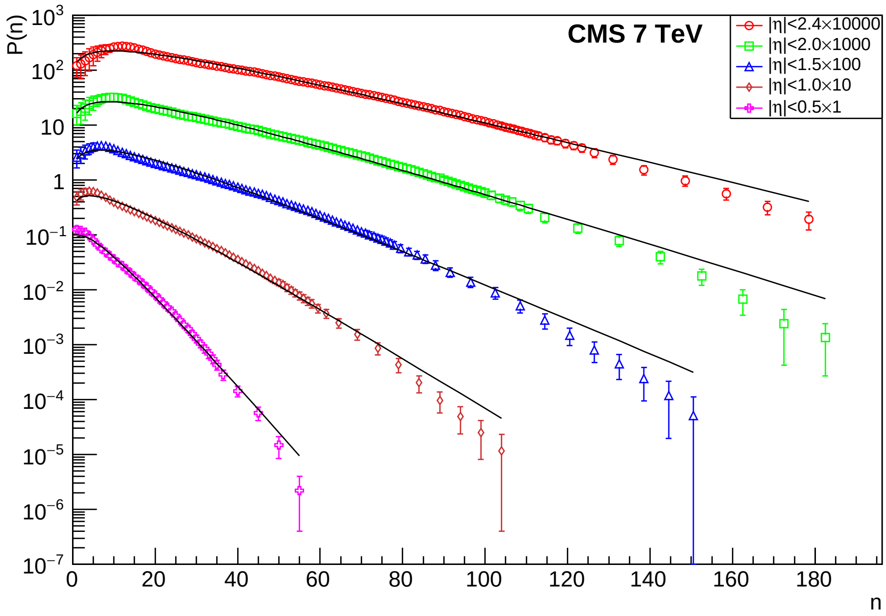

Fig. 4.3 Charged particle multiplicity distribution measured by CMS at $\sqrt{s}$ = 7TeV, fitted with SUSY-included model.

Chapter 5

Conclusion and Further Remarks

A new modified stochastic branching model has been proposed in this thesis. This modified model consists of Simple Birth Process and Poisson Process, and it's a two phase process. The Poisson Process characterized parameter changes from phase one to phase two ($\nu_1 \rightarrow \nu_2$), while Simple Birth Process parameter remains unchanged (λ).

Detailed study about SUSY particle branching shows that the modified model can describe SUSY particle branching process. SUSY particles start branching and later on will decay into ordinary partons and Lightest Supersymmetric Particles (LSP), the ordinary partons will continue to participate branching. This changing of branching sources corresponds to the changing parameter for Poisson Process. The generating function $P_{\text{SUSY}}(x, Y)$ (equation 4.14) and multiplicity distribution $p_{\text{SUSY}}(n, Y)$ (equation 4.15) for pure SUSY particle initiated jets are arrived by direct application of our modified stochastic branching model.

$$P_{\text{SUSY}}(x, Y) = \frac{[(1 - e^{\lambda(Y-Y_1)})x + e^{\lambda(Y-Y_1)}]^{k_1-k_2}}{[(1 - e^{\lambda Y})x + e^{\lambda Y}]^{k_1}} \tag{5.1}$$

$$p_{\mathrm{SUSY}}(n,Y) = e^{\lambda(Y-Y_1)(k_1-k_2)}(1-e^{-\lambda Y})^n(e^{-\lambda Y})^{k_1}\frac{\Gamma(n+k_1)}{\Gamma(k_1)\Gamma(n+1)} \times {}_2F_1(k_2-k_1,-n,1-k_1-n,\frac{e^{\lambda Y}-e^{\lambda Y_1}}{e^{\lambda Y}-1}) \tag{5.2}$$

More general case that initial partons contain both SUSY partons and ordinary partons is also investigated. Based on our understanding about generalized multiplicity distribution (GMD), the GMD generating function is the convolution of pure quarks initiated jets multiplicity distribution(MD) generating function and pure gluon initiated jets MD generating function. Thus, the SUSY plus ordinary parton jets initiated MD generating function $P_{\mathrm{mix}}(x,Y)$ (equation 4.27) is the convolution of squarks, gluinos, quarks initiated jets MD generating function with pure gluon initiated jets MD generating function.

$$\begin{aligned} P_{\mathrm{mix}}(x,Y) &= P_{\tilde{Q}\tilde{G}Q}(x,Y)\times P_G(x,Y) \\ &= \frac{[(1-e^{\lambda(Y-Y_1)})x+e^{\lambda(Y-Y_1)}]^{k_1-k_2}}{[(1-e^{\lambda Y})x+e^{\lambda Y}]^{k_1}}\times\left[\frac{x}{(1-e^{AY})x+e^{AY}}\right]^a \end{aligned} \tag{5.3}$$

The SUSY-included multiplicity distribution (SIMD) $p_{\mathrm{SIMD}}(n,Y)$ (equation 4.28) can be get from generating function.

$$p_{\mathrm{SIMD}}(n,Y) = e^{\lambda(Y-Y_1)(k_1-k_2)}(e^{-\lambda Y})^{k_1+a}(1-e^{-\lambda Y})^{n-a}\frac{\Gamma(n+k_1+a)}{\Gamma(k_1+a)\Gamma(n+1)} \times {}_2F_1[k_2-k_1,-n+a,1-k_1-n,\frac{e^{\lambda Y}-e^{\lambda Y_1}}{e^{\lambda Y}-1}] \tag{5.4}$$

We fit our SUSY-included multiplicity distribution (SIMD) with current pp collision charged particle data at $\sqrt{s}=0.9, 2.36, 7$ TeV at different pseudorapidity intervals, and find the distribution is reduced to generalized multiplicity distribution (GMD) and no SUSY particle participates branching in current collision energy.

This SUSY-included multiplicity distribution (SIMD) would be worth carrying out for further data fitting with higher energy collisions or ultra-high cosmic ray detections. One can also compare this distribution with the Monte Carlo Simulations which include the supersymmetric particles branching.

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Appendix A

Fitting Procedure

Our fitting is based on the minimization of χ^2/dof, the χ^2 is defined as

$$\chi^2 = \sum_n \frac{[p_{\mathrm{data}}(n) - p_{\mathrm{theory}}(n, \{a\})]^2}{\sigma^2} \tag{A.1}$$

Here the $p_{\mathrm{data}}(n)$ represents the experimental data point of multiplicity distribution, while $p_{\mathrm{theory}}(n, a)$ is calculated from theoretical model with $\{a\}$ stands for parameters in the theoretical expression. The σ^2 is the experimental error. When looking at the CMS data, the experimental error consists of systematic error σ_{sys} and statistical error σ_{sta}:

$$\sigma^2 = \sigma_{\mathrm{sys}}^2 + \sigma_{\mathrm{sta}}^2 \tag{A.2}$$

For CMS data, the systematic errors are not symmetric, $\sigma_{\mathrm{sys}}^+ \neq \sigma_{\mathrm{sys}}^-$. In our fitting, we choose the σ_{sys} to be the larger absolute value among σ_{sys}^+ and σ_{sys}^-. The dof is the degree of freedom, here dof $= N_{\mathrm{points}} - N_{\mathrm{para}}$ is the number of data points minus number of parameters. The χ^2/dof is a measure of the goodness of the fitting.

In the standard expression of χ^2, the summation is over integer value of particle number n. However, for CMS data, sometimes the $p_{n'}$ is an averaged value over adjacent multiple bins and n' is not an integer value. In addition, the theoretical expression $p_{\text{theory}}(n, \{a\})$ sometimes only accept integer value n. Considering all above, we use this alternative equation for summation term in χ^2.

$$\frac{[(c-b)p_{\text{data}}(n') - \sum_{n=[b]+1}^{n=[c]} p_{\text{theory}}(n, \{a\})]^2}{[(c-b)\sigma]^2} \tag{A.3}$$

The b and c represent the bin interval boundaries n' lie in, $[b]$ means the biggest integer value less than b.

The biggest difficulty we face in our fitting is the problem with the ordinary Hypergeometric function ${}_2F_1(a, b, c, d)$ in our SUSY-included multiplicity distribution. We use CERN-MINUIT package and ROOT environment via C code. The MathMore library in ROOT contains the Hypergeometric function ${}_2F_1(a, b, c, d)$. However, when the last parameter d in Hypergeometric function goes near 1 (its singularity), the ROOT MathMore Library Hypergeometric function gives us error message. In Mathematica, this problem will not appear. To solve this, we learn how to call the Hypergeometric function of Mathematica from C code and work with ROOT environment at the same time. The CERN-ROOT discussion forum helps us a lot. The commands for running our code are a little bit complicated, since we need to link the Mathematica kernel and ROOT libraries. We list them here.

```
g++ MD_SUSY_7000.cxx -o MD_SUSY_7000 -pthread -std=c++1y -m64
-I/Applications/root_v6.06.00/include -L/Applications
/root_v6.06.00/lib-lGui  -lCore -lRIO -lNet -lHist -lGraf
-lGraf3d -lGpad -lTree -lRint -lPostscript -lMatrix -lPhysics
```

```
-lMathCore -lThread -lMultiProc -lpthread -lm -ldl -lMathmore
-lMinuit -I/Applications/Mathematica.app/SystemFiles/Links
/MathLink/DeveloperKit/MacOSX-x86-64/CompilerAdditions
-L/Applications/Mathematica.app/SystemFiles/Links/MathLink
/DeveloperKit/MacOSX-x86-64/CompilerAdditions -framework
Foundation -lstdc++ -lMLi4 -lMLi3
```

```
./MD_SUSY_7000  -linkmode launch -linkname "/Applications
/Mathematica.app/Contents/MacOS/MathKernel"
```

The software version: Mathematica 10.0.0.0, ROOT 6.06, GCC 5.3.0 Personal PC OS: Mac OS X El Capitan Version 10.11.3. If one is using other operating system (OS), Linux or Windows, the commands need to be changed accordingly.

Appendix B

C Program

Here we attach the data processing and fitting C programs , with the files needed by this program. This is for CMS $\sqrt{s} = 7$ TeV and $|\eta| < 2.4$, for other energy and pseudorapidity range one just modifies the numbers and filenames in the code.

B.1 Readfile

Delete the first point p_0 in raw data, and calculate σ from σ_{sys} and σ_{sta}. The raw data is attached behind the C program.

```
//readfile_7000.c
#include <stdio.h>
#include <math.h>
#include "nr.h"
#include "nrutil.h"

#define HEIGHT 126
```

```
int main()
{
  FILE *fp, *fw, *fl;// *fwa;
 double my_varible, tempt, *x, *xlow, *xhigh, *y, *dy1p, *dy1m, *dy2p,\
         *dy2m, *sig;
  double sum=0;
  int i,j;

  x = vector(1,HEIGHT);
  xlow = vector(1,HEIGHT);
  xhigh = vector(1,HEIGHT);
  y = vector(1,HEIGHT);
  dy1p = vector(1,HEIGHT);
  dy1m = vector(1,HEIGHT);
  dy2p = vector(1,HEIGHT);
  dy2m = vector(1,HEIGHT);
  sig = vector(1,HEIGHT);

  fp = fopen("Data_7000_2.4.txt", "r");
  fw = fopen("Data_7000_2.4_trans.txt", "w");
  fl = fopen("Data_7000_2.4_plot.txt", "w");
  //fwa = fopen("Data_7000_1.5_mathmatica.dat", "w");

for(i=1;i<=1;i++)
{
```

```
  fscanf(fp,"%lf", &my_varible);
  fscanf(fp,"%lf", &my_varible);
  fscanf(fp,"%lf", &my_varible);
  fscanf(fp,"%lf", &my_varible);
  fscanf(fp,"%lf", &my_varible);
  fscanf(fp,"%lf", &my_varible);
  fscanf(fp,"%lf", &my_varible);
  fscanf(fp,"%lf", &my_varible);
}


for(i=1;i<=HEIGHT;i++)
{
  fscanf(fp,"%lf", &my_varible);
  x[i]= my_varible;
  fscanf(fp,"%lf", &my_varible);
  xlow[i]= my_varible;
  fscanf(fp,"%lf", &my_varible);
  xhigh[i]= my_varible;
  fscanf(fp,"%lf", &my_varible);
  y[i] = my_varible;
  if ( (xhigh[i]-xlow[i])== 1.0 )
  {
    //printf("%15f \n", x[i]);
     sum = sum + y[i];
   }
   else
```

```
  {
     sum = sum + y[i]*(xhigh[i]-xlow[i]);
   }
   fscanf(fp,"%lf", &my_varible);
   dy1p[i] = my_varible;
   fscanf(fp,"%lf", &my_varible);
   dy1m[i] = my_varible;
   fscanf(fp,"%lf", &my_varible);
   dy2p[i] = my_varible;
   fscanf(fp,"%lf", &my_varible);
   dy2m[i] = my_varible;
   sig[i] = sqrt(pow(dy1p[i],2) + pow(dy2p[i],2));
 }

  printf("sum =%.15f \n ", sum);

  for(i=1;i<=HEIGHT;i++)
  {
   fprintf(fw,"%.15f ", x[i]);
   fprintf(fl,"%.15f ", x[i]);
   printf("%.15f  ", x[i]);
   fprintf(fw,"%.15f  ", xlow[i]);
   printf("%.15f  ", xlow[i]);
   fprintf(fw,"%.15f  ", xhigh[i]);
   printf("%.15f  ", xhigh[i]);
   tempt = y[i] ;
```

```
 y[i] = tempt/sum;
 fprintf(fw,"%.15f  ", y[i]);
 fprintf(fl,"%.15f  ", y[i]);
 printf("%.15f  ", tempt);
 printf("%.15f  ", y[i]);

 tempt = sig[i];
 sig[i] = tempt/sum;

 fprintf(fw,"%.15f  \n", sig[i]);
 fprintf(fl,"%.15f  \n", sig[i]);
 printf("%.15f  ", tempt);
 printf("%.15f  \n", sig[i]);
}



fclose(fp);
fclose(fw);
fclose(fl);
free_vector(x,1,HEIGHT);
free_vector(xlow,1,HEIGHT);
free_vector(xhigh,1,HEIGHT);
free_vector(y,1,HEIGHT);
free_vector(dy1p,1,HEIGHT);
free_vector(dy1m,1,HEIGHT);
free_vector(dy2p,1,HEIGHT);
```

```
    free_vector(dy2m,1,HEIGHT);
    return 0;
}
```

The raw data of CMS $\sqrt{s} = 7$TeV and $|\eta| < 2.4$.

```
0.0 -0.5 0.5 0.0494 +0.0063 -0.0063 +0.0269 -0.0304
1.0 0.5 1.5 0.01789 +8.0E-4 -8.0E-4 +0.00373 -0.00244
2.0 1.5 2.5 0.01895 +3.7E-4 -3.7E-4 +0.00438 -0.00379
3.0 2.5 3.5 0.02174 +4.3E-4 -4.3E-4 +0.00533 -0.00491
4.0 3.5 4.5 0.0255 +4.1E-4 -4.1E-4 +0.00586 -0.00546
5.0 4.5 5.5 0.02961 +4.0E-4 -4.0E-4 +0.00588 -0.00543
6.0 5.5 6.5 0.0335 +4.3E-4 -4.3E-4 +0.00543 -0.00491
7.0 6.5 7.5 0.03646 +4.3E-4 -4.3E-4 +0.00471 -0.00414
8.0 7.5 8.5 0.03826 +4.2E-4 -4.2E-4 +0.00401 -0.00338
9.0 8.5 9.5 0.03906 +4.2E-4 -4.2E-4 +0.0037 -0.00307
10.0 9.5 10.5 0.0392 +4.3E-4 -4.3E-4 +0.00355 -0.00295
11.0 10.5 11.5 0.03868 +4.2E-4 -4.2E-4 +0.00337 -0.00278
12.0 11.5 12.5 0.03753 +4.0E-4 -4.0E-4 +0.00318 -0.00261
13.0 12.5 13.5 0.03596 +4.0E-4 -4.0E-4 +0.003 -0.00246
14.0 13.5 14.5 0.0341 +3.9E-4 -3.9E-4 +0.00282 -0.00232
15.0 14.5 15.5 0.03206 +3.9E-4 -3.9E-4 +0.00264 -0.00213
16.0 15.5 16.5 0.03 +3.8E-4 -3.8E-4 +0.00244 -0.00193
17.0 16.5 17.5 0.02802 +3.7E-4 -3.7E-4 +0.00224 -0.00175
18.0 17.5 18.5 0.02616 +3.6E-4 -3.6E-4 +0.00205 -0.00157
19.0 18.5 19.5 0.02448 +3.6E-4 -3.6E-4 +0.00187 -0.00141
20.0 19.5 20.5 0.023 +3.5E-4 -3.5E-4 +0.00172 -0.00128
21.0 20.5 21.5 0.0217 +3.3E-4 -3.3E-4 +0.00161 -0.00118
```

```
22.0 21.5 22.5 0.02058 +3.2E-4 -3.2E-4 +0.00152 -0.00112
23.0 22.5 23.5 0.01959 +3.2E-4 -3.2E-4 +0.00144 -0.00107
24.0 23.5 24.5 0.01866 +3.1E-4 -3.1E-4 +0.00137 -0.00103
25.0 24.5 25.5 0.01776 +3.1E-4 -3.1E-4 +0.00129 -9.8E-4
26.0 25.5 26.5 0.01685 +3.1E-4 -3.1E-4 +0.00121 -9.2E-4
27.0 26.5 27.5 0.01595 +3.0E-4 -3.0E-4 +0.00113 -8.7E-4
28.0 27.5 28.5 0.01504 +3.0E-4 -3.0E-4 +0.00108 -8.3E-4
29.0 28.5 29.5 0.01415 +2.9E-4 -2.9E-4 +0.00111 -8.0E-4
30.0 29.5 30.5 0.01326 +2.9E-4 -2.9E-4 +9.5E-4 -7.7E-4
31.0 30.5 31.5 0.01241 +2.8E-4 -2.8E-4 +9.1E-4 -7.6E-4
32.0 31.5 32.5 0.01161 +2.8E-4 -2.8E-4 +8.7E-4 -7.4E-4
33.0 32.5 33.5 0.01087 +2.8E-4 -2.8E-4 +8.3E-4 -7.2E-4
34.0 33.5 34.5 0.01015 +2.7E-4 -2.7E-4 +7.9E-4 -6.8E-4
35.0 34.5 35.5 0.00948 +2.6E-4 -2.6E-4 +7.3E-4 -6.3E-4
36.0 35.5 36.5 0.00887 +2.5E-4 -2.5E-4 +6.8E-4 -5.9E-4
37.0 36.5 37.5 0.00829 +2.5E-4 -2.5E-4 +6.2E-4 -5.4E-4
38.0 37.5 38.5 0.00777 +2.4E-4 -2.4E-4 +5.6E-4 -5.1E-4
39.0 38.5 39.5 0.00734 +2.4E-4 -2.4E-4 +5.2E-4 -5.1E-4
40.0 39.5 40.5 0.00681 +2.3E-4 -2.3E-4 +4.9E-4 -4.8E-4
41.0 40.5 41.5 0.00624 +2.2E-4 -2.2E-4 +4.8E-4 -4.4E-4
42.0 41.5 42.5 0.00578 +2.1E-4 -2.1E-4 +4.7E-4 -4.3E-4
43.0 42.5 43.5 0.00535 +2.0E-4 -2.0E-4 +4.6E-4 -4.3E-4
44.0 43.5 44.5 0.00494 +2.0E-4 -2.0E-4 +4.5E-4 -4.2E-4
45.0 44.5 45.5 0.00457 +1.9E-4 -1.9E-4 +4.4E-4 -4.0E-4
46.0 45.5 46.5 0.00422 +1.9E-4 -1.9E-4 +4.3E-4 -3.8E-4
47.0 46.5 47.5 0.00388 +1.9E-4 -1.9E-4 +4.0E-4 -3.5E-4
```

```
48.0 47.5 48.5 0.00356 +1.8E-4 -1.8E-4 +3.8E-4 -3.2E-4
49.0 48.5 49.5 0.00326 +1.7E-4 -1.7E-4 +3.5E-4 -2.9E-4
50.0 49.5 50.5 0.00298 +1.6E-4 -1.6E-4 +3.3E-4 -2.7E-4
51.0 50.5 51.5 0.00274 +1.6E-4 -1.6E-4 +3.1E-4 -2.7E-4
52.0 51.5 52.5 0.00253 +1.5E-4 -1.5E-4 +2.9E-4 -2.8E-4
53.0 52.5 53.5 0.00234 +1.5E-4 -1.5E-4 +2.7E-4 -3.0E-4
54.0 53.5 54.5 0.00212 +1.4E-4 -1.4E-4 +2.7E-4 -3.0E-4
55.0 54.5 55.5 0.00189 +1.3E-4 -1.3E-4 +2.7E-4 -2.7E-4
56.0 55.5 56.5 0.00167 +1.3E-4 -1.3E-4 +2.6E-4 -2.3E-4
57.0 56.5 57.5 0.0015 +1.3E-4 -1.3E-4 +2.3E-4 -2.0E-4
58.0 57.5 58.5 0.00136 +1.2E-4 -1.2E-4 +2.1E-4 -1.8E-4
59.0 58.5 59.5 0.00123 +1.1E-4 -1.1E-4 +1.8E-4 -1.6E-4
60.0 59.5 60.5 0.00112 +1.0E-4 -1.0E-4 +1.7E-4 -1.3E-4
61.0 60.5 61.5 0.001041 +9.3E-5 -9.3E-5 +1.47E-4 -1.2E-4
62.5 61.5 63.5 9.17E-4 +7.3E-5 -7.3E-5 +1.25E-4 -1.42E-4
64.5 63.5 65.5 7.64E-4 +5.8E-5 -5.8E-5 +1.06E-4 -1.27E-4
67.5 65.5 69.5 4.92E-4 +3.2E-5 -3.2E-5 +1.05E-4 -9.2E-5
74.5 69.5 79.5 2.45E-4 +2.0E-5 -2.0E-5 +6.7E-5 -6.5E-5
84.5 79.5 89.5 9.9E-5 +1.3E-5 -1.3E-5 +3.3E-5 -3.5E-5
94.5 89.5 99.5 1.64E-5 +4.1E-6 -4.1E-6 +8.1E-6 -8.5E-6
```

B.2 Fitting and Plotting

The fitting program for CMS $\sqrt{s} = 7$ TeV and $|\eta| < 2.4$, the files it needed have been generated by readfile program.

```
// MD_SUSY_7000.cxx
```

```
#include "/Applications/Mathematica.app/SystemFiles/Links/MathLink\
/DeveloperKit/MacOSX-x86-64/CompilerAdditions/mathlink.h"
#include <stdio.h>
#include "TCanvas.h"
#include "TMath.h"
#include "TROOT.h"
#include "TGraphErrors.h"
#include "TF1.h"
#include "TLegend.h"
#include "TLatex.h"
#include "TRandom.h"
#include "RooNumber.h"
#include "TGraph.h"
#include "TMinuit.h"



#define height 126
#define n_aver 30.4

//mathematica
MLENV env;
MLINK my_link;

Double_t my_x[height], my_xlow[height], my_xhigh[height],\
my_y[height], my_errory[height];
```

```
double hyper(double a, double b, double c, double d)
{
  int packet;
  double tem;

  MLPutFunction(my_link, "Hypergeometric2F1", 4);
  MLPutReal64(my_link, a);
  MLPutReal64(my_link, b);
  MLPutReal64(my_link, c);
  MLPutReal64(my_link, d);
  MLEndPacket(my_link);

  /* get packets until we find a ReturnPacket or error */
 while ((packet = MLNextPacket(my_link)) && packet != RETURNPKT)
  MLNewPacket(my_link);

  if (MLError(my_link))
  printf("error.\n");
  else
  {
    double result;
    MLGetReal64(my_link, &result);
    /* we know that the result is an integer in this case */
    tem = result;
  }
```

```
  return tem;
}



Double_t func(Double_t x,Double_t *a)
/* a[0]=m, a[1]= 1 - m , a[2]= k1, a[3]=k2-k1, a[4] = a*/
{
  Double_t f,g,h,k;

  f = a[3];
  g = -x+a[4];
  h = 1 - a[2] - x;
  k = a[1]/(a[0]+a[1]-1);

 Double_t value = pow((a[0]+a[1])/a[0], -a[3])*pow(1-1/(a[0]+a[1]),x-a[4])
 *pow(1/(a[0]+a[1]), a[2])/(x-a[4])/TMath::Beta(x-a[4],a[2]+a[4])\
  *hyper(f,g,h,k);

  return value;
}

void fcn(Int_t &npar, Double_t *gin, Double_t &f, Double_t *a, Int_t iflag)
{
  Int_t i,j;

  //calculate chisquare
```

```
  Double_t chisq = 0;

  Double_t delta, tempt1=0;


  for (i=0;i<height; i++)

  {

   if( (my_xhigh[i]-my_xlow[i])== 1.0 && (Int_t)my_x[i]== my_x[i] )

    {

      delta  = (my_y[i]-func(my_x[i],a))/my_errory[i];

      chisq += delta*delta;

    }

    else

    {

      tempt1 = my_y[i]*(my_xhigh[i]-my_xlow[i]);

      for(j=my_xlow[i]+0.5;j<my_xhigh[i];j++)

      {

        tempt1 = tempt1 - func(j,a);

      }

      delta = tempt1/((my_xhigh[i]-my_xlow[i])*my_errory[i]);

      chisq += delta*delta;

    }

  }

   f = chisq;

}


 void MD_SUSY_7000()

 {
```

```
FILE *fp, *fw;
int i,j;
Double_t my_varible;
Double_t xth[height], yth[height];

fp = fopen("Data_7000_2.4_trans.txt", "r");
fw = fopen("Data_7000_2.4_SUSY_theory.txt", "w");

for(i=0;i<height;i++)
{
  fscanf(fp,"%lf", &my_varible);
  my_x[i]= my_varible;
  fscanf(fp,"%lf", &my_varible);
  my_xlow[i] = my_varible;
  fscanf(fp,"%lf", &my_varible);
  my_xhigh[i] = my_varible;
  fscanf(fp,"%lf", &my_varible);
  my_y[i]= my_varible;
  fscanf(fp,"%lf", &my_varible);
  my_errory[i]= my_varible;
 }

 Int_t parN=5 ;// parameter number
 TMinuit *gMinuit = new TMinuit(parN);
 //initialize TMinuit with a maximum of 5 points
 gMinuit->SetFCN(fcn);
```

```
Double_t arglist[10];
Int_t ierflg = 0;

arglist[0] = 1;
gMinuit->mnexcm("SET ERR", arglist ,1,ierflg);

static Double_t vstart[5];/*={13.481189992282294
    ,0.000034166743607
    ,1.584730790059435
    ,0.304155044507726
    ,0.000000000559808};*/

TRandom r0(0);



vstart[0] = 1+10*r0.Rndm();
vstart[1] = 10*r0.Rndm();
vstart[2] = 10*r0.Rndm();
vstart[3] = 10*r0.Rndm();
vstart[4] = 10*r0.Rndm();

/* a[0]=m, a[1]= l - m , a[2]= k1, a[3]=k2-k1, a[4] = a*/
static Double_t step[5] = {0.01 , 0.01, 0.01, 0.01, 0.01};
gMinuit->mnparm(0, "m", vstart[0], step[0], 1,100,ierflg);
gMinuit->mnparm(1, "l-m", vstart[1], step[1], 0,100,ierflg);
```

```
gMinuit->mnparm(2, "k1", vstart[2], step[2], 0,100,ierflg);
gMinuit->mnparm(3, "k2", vstart[3], step[3], 0,100,ierflg);
gMinuit->mnparm(4, "a", vstart[4], step[4], 0,100,ierflg);

// Now ready for minimization step
arglist[0] = 4000;   //max iteration
arglist[1] = 0.01;    //related to errors, tollerance
gMinuit->mnexcm("MINImize", arglist ,2,ierflg);
//MIGRAD,SIMPLEX,MINImize,SEEk

// Print results
Double_t amin,edm,errdef;
Int_t nvpar,nparx,icstat;
gMinuit->mnstat(amin,edm,errdef,nvpar,nparx,icstat);
// amin, the best function value so far
// edm, the estimated vertical distance to minimum
// errdef, the uncertainty for parameters
// nvpar, number of currently variable parameters
// nparx,the highest (external) parameter number defined by user
// icstat, a status integer indicating how good is the covariance

printf("chi2/dof minimum =%.15f \n ", amin/(height-parN));

Double_t parth[parN], curV, curE;

printf("~~~~~~~~~ Parameter final values ~~~~~~~~~~~~~~~\n");
```

```
// -- Extract the fit parameters from minuit
for(i=0;i<parN;i++)
{
  gMinuit->GetParameter(i, curV, curE);
  parth[i] = curV;
  printf("%.15f \n ", parth[i]);
}

printf("~~~~~~~~~ Original values ~~~~~~~~~~~~~~~~~~~~~\n");
printf("m=%.15f \n ", parth[0]);
printf("l=%.15f \n ", parth[0]+parth[1]);
printf("k1=%.15f \n ", parth[2]);
printf("k2=%.15f \n ", parth[2]+parth[3]);
printf("a=%.15f \n ", parth[4]);

Double_t mean=0, mean1=0, mean2=0, mean3=0;
Double_t tempt2;
for(Int_t i=0; i<height; i++)
{
if( (my_xhigh[i]-my_xlow[i])== 1.0 && (Int_t)my_x[i]== my_x[i] )
  {
    xth[i]= my_x[i];
    fprintf(fw,"%.15f   ", xth[i]);
    yth[i]= func(xth[i],parth);
    fprintf(fw,"%.15f   \n", yth[i]);
    mean = mean + xth[i]*yth[i];
```

```
}
else if ( my_x[i]-(Int_t)my_x[i] == 0.5)
{
  xth[i]= my_x[i];
  fprintf(fw,"%.15f   ", xth[i]);


  tempt2 = 0;
  for(j=my_xlow[i]+0.5;j<my_xhigh[i];j++)
  {
    tempt2 = tempt2 + func(j,parth);
    mean = mean + j*func(j,parth);
  }
  yth[i] = tempt2/(my_xhigh[i]-my_xlow[i]);
   fprintf(fw,"%.15f   \n", yth[i]);


}
else
{
  xth[i] = my_x[i];
  fprintf(fw,"%.15f   ", xth[i]);
  tempt2 = 0;
  tempt2 = (Int_t) my_x[i];
  Double_t delta =0;
  delta = my_x[i] - tempt2;
yth[i] = (1-delta)*func(tempt2,parth) + delta*func(tempt2+1,parth);
  fprintf(fw,"%.15f   \n", yth[i]);
```

```
    tempt2 = 0;
    for(j=my_xlow[i]+0.5;j<my_xhigh[i];j++)
    {
      tempt2 = tempt2 + func(j,parth);
      mean = mean + j*func(j,parth);
    }
   }
  }
  printf("mean = %.15f   \n", mean);

printf("~~~~~~~~~~~~ Parameter initial values ~~~~~~~~~~~~~~~\n");
  for(i=0;i<parN;i++)
  {
    printf("%.15f \n ", vstart[i]);
  }



   // Plot the data with and theory function
   // The canvas on which we'll draw the graph
  TCanvas* mycanvas = new TCanvas();
  gPad->SetLogy(1);

  // The values and the errors on the Y axis
TGraphErrors graph_error("./Data_7000_2.4_plot.txt","%lg %lg %lg");
  graph_error.SetTitle(";n;P(n)");
```

```
  //Draw the error band
  graph_error.SetFillColor(kYellow);
  graph_error.SetMinimum(0.00001);
  graph_error.SetMaximum(0.1);
  graph_error.DrawClone("E3AL");// E3 draws the band


TGraphErrors graph("./Data_7000_2.4_plot.txt","%lg %lg %lg");
  graph.SetMarkerStyle(4);
  graph.SetMarkerColor(kBlack);
  graph.SetMarkerSize(0.3);
  graph.SetMinimum(0.00001);
  graph.SetMaximum(0.1);
  graph.DrawClone("PESame");
  //L impose the drawing of the graphs' line
  //E impose the drawing of the graphs' error bars
  //P impose the drawing of the graphs'


  TGraph graph_th(height, xth, yth);
  graph_th.SetLineStyle(1);
  graph_th.SetLineColor(2);
  graph_th.SetLineWidth(3);
  graph_th.SetMinimum(0.00001);
  graph_th.SetMaximum(0.1);
  graph_th.DrawClone("LSame");
```

```
    TLegend leg(.75,.75,.9,.9);
    leg.SetFillColor(0);
    graph.SetFillColor(0);
    graph_th.SetFillColor(0);
    leg.AddEntry(&graph,"Data","lep");
    leg.AddEntry(&graph_th,"SUSY");
    leg.DrawClone("Same");

   TLatex text(120,0.05,"#scale[0.8]{#splitline{CMS 7 TeV}{|#eta|<2.4}}");
    text.DrawClone();

    mycanvas->Print("MD_SUSY_CMS_7000_24.pdf");
    mycanvas->Print("MD_SUSY_CMS_7000_24.eps");
    mycanvas->Print("MD_SUSY_CMS_7000_24.png");

}

 #ifndef __CINT__

 int main(int argc, char *argv[])
 {
   int errno;
   env = MLInitialize(0);
   my_link = MLOpenArgcArgv(env, argc, argv, &errno);
   MLActivate(my_link);
```

```
  MD_SUSY_7000();


  MLClose(my_link);

  MLDeinitialize(env);

  return 0;

 }


#endif
```